UPPSALA UNIVERSITET
NATURGEOGRAFISKA
INSTITUTIONEN
AVDELNINGEN FÖR HYDROLOGI

ANDERS WILLÉN

A STOCHASTIC HYDROLOGY APPROACH TO THE OPERATION OF SURFACE WATER RESERVOIRS

Part:I Stochastic short term modelling of inflow and runoff

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PREFACE

The topic of this thesis was suggested to me by my supervisor professor Erik Eriksson, who has given me much valuable guidance and with whom I have had many inspiring discussions. The data values from Korsvattnet and Stora Mjölkvattnet were received from Indalsälvens Vattenregleringsföretag (The River Indalsälven Regulation Company), where I have had valuable discussions with several engineers, special credit is given to Eng. Hilding Hedqvist.

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Uppsala, May 1978

Anders Willén

ABSTRACT

A STOCHASTIC HYDROLOGY APPROACH TO THE OPERATION OF SURFACE WATER RESERVOIRS (In English).

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This thesis deals with stochastic modelling in hydrology applied to the operation of multi-reservoir systems. The un-predictability of future inflows is a fundamental feature of this problem and therefore this is a problem of stochastic optimization. Stochastic models of inflow and runoff are developed and tested in various ways. Seasonality is very marked in the records used in this study, with high flows in spring owing to snow-melt, and low flows in winter owing to snow accumulation. Two kinds of models are used for the non-spring seasons. In the Rec-Ris model, the idea is to take into account the quite different mechanisms involved during the recession and rising parts of runoff. The other model is based on conditional probability distributions, and it is applied to the inflow at two sites in central Sweden. Here the probability distribution of next inflow is conditioned by present inflow. In the spring, one model first determines the total amount of inflow during some weeks and then this amount is distributed over these weeks. In the other model we first make a prediction of total runoff during some pentades (pentade = five days) in the spring, and then generate the runoff pentade by pentade based on this prediction. Our stochastic models of inflow are applied in the optimization of a 5-reservoir system regulated for hydropower production. This is done by Monte-Carlo generations of synthetic series, from which probabalistic forecasts of future inflow are derived.

A chance-constrained optimization model is then applied to the operation of our reservoir system. In this model the operation is determined for the whole system for a number of weeks ahead, maximum 52 weeks ahead. Thereafter a simulation model is applied, where the real inflow is used as input, and some changes in the releases are made. Using the state derived from this simulation model and new forecasts of inflow, re-optimization is applied to the rest of the planning period, and the simulation model is applied, etc.,

(KEY WORDS: Inflow modelling; Runoff modelling; Monte-Carlo simulation; Stochastic optimization; Multi-reservoir systems; Stochastic modelling).

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1 INTRODUCTION

This report is part one of a Ph.D. thesis on stochastic modelling and optimization in hydrology. Here we deal with stochastic modelling with emphasis on short term models and in part two (Willén 1978) we deal with stochastic optimization. The purpose is to use stochastic models of availability of water in optimization of the operation of water resources systems, specially hydro-electricity systems.

In this paper we deal with both inflow and runoff. Let us discuss the concept of inflow.

In this thesis inflow means local net inflow, and it is defined for a regulated reservoir by the equation below.

$$S_t = S_{t-1} + Q_t - X_t + XU_t$$

S_t Storage in the regulated reservoir at end of time step t.
Q_t Inflow during time step t.
X_t Release from the reservoir during time step t.
XU_t Total amount of upstream release that enters the reservoir during time step t.

From this equation it is obvious that Q is not equal to the sum of water entering the reservoir by the river(s). Instead it is the net effect of this sum, evaporation and groundwater inflow (which may be positive or negative).

By using equations like this in reservoir operation studies, we try to separate the behaviour of reservoirs into two distinct categories; the natural process, Q, and the human controllable, X and XU. This decomposition between man and nature is not without objections. In reality large storage may increase evaporation and seepage to groundwater (or decrease groundwater inflow), which means that Q decreases with increasing storage. Thus inflow is not independent of X and XU. In arid regions the evaporation effect may be considerable and make the decoupling between man and nature impossible.

But in the humid climate of the Swedish mountains it is reasonable to consider Q independent of X and XU. This approximation is made throughout this thesis.

The next two chapters are devoted to general discussions on mathematical and stochastic models respectively. Chapter 4 deals with modelling of pentade (five-day mean) runoff at Ytterholmen and Chapter 5 with weekly inflows to Korsvattnet and Stora Mjölkvattnet.

As we are discussing a large number of quite different models, it was not considered reasonable to maintain notational consistency throughout the thesis. However in the chapters dealing with the author´s own models and methods notational consistency is maintained. This should not present any difficulties to the reader, as the symbols are defined whereever confusion may appear.

The stochastic models of inflow are used in part two of this thesis (Willén 1978) to optimize the release of a hydro-electricity system with 5 reservoirs.

2 SOME GENERAL ASPECTS OF MATHEMATICAL MODELS IN HYDROLOGY

Although the emphasis in this report is on stochastic models of runoff and reservoir inflow, a short general discussion on mathematical models is included. No attempt is made to give a complete review and the references in this chapter are chosen from Sweden only.

Three main approaches exist in this field.

1) Strict physical approach based on the laws of hydrodynamics, thermodynamics etc.
2) Input-output modelling, where input usually is weather and output may be runoff.
3) Stochastic models.

Unless otherwise stated, when talking about "Stochastic Processes" in this report we mean processes that give a probabalistic relation between the future values of a vector Θ and its past history (including its present value). The models based on strict physical laws look theoretically sound, but there are many critical points. Most physical laws (in e.g. hydrodynamics) are formulated as partial differential equations with appropriate boundary conditions. Even if all parameters of the process can be determined from the laws of physics or measurements with high precision, the boundary conditions must be determined by one or another kind of measurements and to some degree of precision. This is necessary since without these boundary conditions we cannot claim to have an exact physical model. And often (e.g. in processes influenced by turbulence) the physical laws are not perfectly known, so there remains process parameters to be estimated from measurements.

Anyhow, the strict physical approach requires intense measurements in order to determine the boundary conditions and maybe some parameters. And in theory these measurements should be made over a continuous domain (space coordinates, time etc.). Another problem is the differential equations which in most cases cannot be solved analytically but must be treated by numerical methods. These methods generally include discretisizing the continuous domain (over which the equations are defined) in one way or another. The strict physical approach favours the use of very small increments in the discretized domain, but this may lead to computational infeasibility. And if we use rather large increments we may leave the strict physical approach and enter input-output modelling.

A few examples of problems often attacked by the physical approach are:

Circulation in lakes (Bengtsson 1973).
Unsteady flows in rivers (Sjöberg 1976).
Evaluation of groundwater resources by pumping (Gustavsson 1977, Andersson 1976).

As an example of input-output models see Fig 1 below.

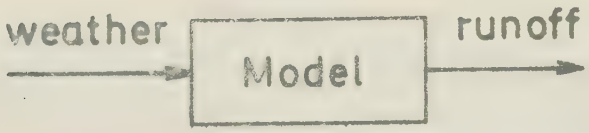

Fig 1 Input-output model.

Such models are often used. The models often consist of several boxes such that the output from one box is the input to another box etc., i.e. Fig 1 is a very coarse simplification of real models. The input mainly consists of precipitation but often temperature, wind etc. are also included. These models may range from strict physical models (as in approach 1 above) in extreme cases to pure statistical analysis of relations between input and output without phenomenological arguments. Usually one use physical arguments to some extent, but the complete set of equations defining the model is very seldom derived exclusively from basic laws of physics.

The HBV-model developed by Bergström (1976) is a weather-runoff model which includes snow-melt routines. These models are nearly always deterministic in the sense that a given weather applied to a certain history of runoff results in one and only development of future runoff. It is also possible to build stochastic models, where the same input may give rise to several traces of output, but parameter estimation for such models may be difficult.

Gottschalk and Nordberg (1977) applied input-output models with groundwater levels as output and weather as input. The third approach is the main subject of this report and it will be discussed in the following chapters.

If we have a perfect strictly physical model, where all the basic natural laws are well known (including its parameters), this model has the important merit of universality. This means that the model is easily (in theory) applied to different geographical sites. But the approximations often needed to solve the equations and incomplete knowledge of some basic laws may reduce the general applicability.

One purpose of models is forecasting of future runoff. Both the physical approach and input-output models require some forecasting of future weather,

but this is not needed in stochastic models. Weather forecasting with confidence is impossible for lead times longer than one week, and this is the weakest point when forecasting with these kinds of models. When using stochastic models for this purpose one computes probabilities of various future streamflow sequences. In a few cases this can be done with analytical methods, but in most cases we have to apply Monte-Carlo methods. In Monte-Carlo methods future runoff are generated according to the laws of chance defining the model by using "random numbers". Examples of such generating schemes will be given later in this paper.

One possibility is to develop a stochastic model of the input to an input-output model, generate traces of future weather by Monte-Carlo methods and compute probabilities of future runoff by feeding these Monte-Carlo generated inputs into the model. This approach looks promising even if it has some drawbacks.

i) The input is often multidimensional and generation of multidimensional sequences is generally more time consuming than generation of univariate ones. One alternative is to use traces of the historic weather record as input, but this requires the existence of a long record.

ii) The runoff obtained in this way does not have the same value as the real runoff, as no model is perfect.

Argument ii) means that probabilities of future runoff computed in this way may be less reliable than those computed directly from the stochastic process of runoff, provided the historic series of runoff is long enough to enable reasonable success in parameter estimation and model building.

Whereas a strict physical model mainly requires intense measurements, the stochastic approach requires measurements over a long period of time.

The author thinks the use of input-output models or physical models is to be prefered in short-term forecasting, because these methods often make use of both flow conditions and information on the weather. But if long-term forecasting is desired, stochastic models are probably better.

3 STOCHASTIC MODELLING OF INFLOW AND RUNOFF: A REVIEW AND SOME GENERAL ASPECTS

3.1 GENERAL ASPECTS

In the following we sometimes discuss inflow and sometimes runoff, but in stochastic model building these two phenomena are very similar. This means that what is said in the following concerns both inflow and runoff unless the contrary is explicitly stated.

We distinguish between seasonal modelling and annual modelling, where seasonal may mean quarterly, monthly, daily, etc.

Characteristic of runoff in Sweden is the seasonality in the probability distributions and often also in the persistence, see for example Gottschalk (1975). Short term modelling without taking seasonality into account is impossible. See tables 2, 3, 12-15 for some parameters of inflow and runoff at some sites in Sweden.

Amongst the several approaches in stochastic modelling of seasonal runoff we can distinguish two main ones. Although we discuss explicitly weekly inflows a change to e.g. monthly inflows involves no difficulties.

In the first approach we try to somehow construct our model on the 52 weekly probability distributions of inflows, or at least some parameters of these distributions (e.g. the means and standard deviations of inflows or some transformations of inflows).

In the second approach we try to build the stochastic model on some mechanisms govering the process rather than thinking of preserving characteristics of the weekly distributions. These mechanisms may be physical or probabalistic. Of course a good model designed according to this idea should to some extent preserve characteristics of the weekly distributions also, but this is not used in model building and possibly not even in parameter estimation. It may, however, be a useful testing tool.

Let us compare the two approaches! The historic series will not repeat itself and future series will have different weekly distributions, but the governing mechanisms will probably remain rather intact in the near future. However, we do not know the basic inflow processes perfectly, and the parameters of the mechanisms probably possess sampling variations of similar kind as the weekly distributions. Moreover, the phenomena

involved are extremely complicated, so which approach to use is a matter of discussion. Some models can be considered a combination of the two approaches.

3.2 MODELS BASED ON SEASONAL DISTRIBUTIONS

First the notations used.

$Q_{i,j}$ Inflow during week j year i.

u_j Mean of inflow week j.

s_j Standard deviation of inflow week j.

$$Y_{i,j} = (Q_{i,j} - u_j)/s_j \tag{1}$$

$E_{i,j}$ A sequence of independent normally distributed random numbers with mean equal to zero and standard deviation equal to one.

ρ_j One step auto correlation coefficient (correlation between $Y_{i,j}$ and $Y_{i,j-1}$) for Y.

Here and in the sequel it is understood that if $j < k$ then (i,j-k) means (i-1, j-k+52).

Examples:

$$Y_{i,j} = \rho_j \cdot Y_{i,j-1} + \sqrt{1-\rho_j^2} \cdot E_{i,j} \tag{2}$$

which defines a first order autoregressive process.

$$Y_{i,j} = \sum_{k=1}^{m} B_{j,k} \cdot Y_{i,j-k} + \sigma \cdot E_{i,j} \tag{3}$$

which is an autoregressive model of order m.

$B_{j,k}$ Autoregressive coefficients.

σ A constant.

Equations (2) and (3) imply normally (Gaussian) distributed Y and Q. The normal distribution with zero mean and unit standard deviation is defined by

$$F_z(Z) = \text{Prob}\,(z \leq Z) = (1/\sqrt{2\pi}) \cdot \int_{-\infty}^{Z} \text{Exp}(-t^2/2)dt \tag{4}$$

and it is called the "unit normal distribution". If a variable x has the probability distribution G(x), then for any value x_1 of x there exists one and only one value z_1 of z such that

$$G(x_1) = F(z_1) \tag{5}$$

z_1 is called "the corresponding unit normal value of x_1" in this report and this transformation is applied on occasions in this paper.

If $Y_{i,j}$ has the distribution defined by eq (4), then $Q_{i,j}$ defined by eq (3) is Gaussian distributed with mean equal to u_j and standard deviation equal to s_j.

Generation of synthetic series corresponding to model (3) is simple (and model (2) is a special version of (3)).

i) Generate a sequence of normally distributed and independent random numbers E.
ii) Obtain Y from eq (3).
iii) Finally Q is obtained from eq (1).

In order to follow this scheme we must choose m starting values of Y, but this choice is often not critical. If the model is used in probabalistic forecasting we choose the latest known values.

But the inflow is very seldom normally distributed and therefore models (2) or (3) are often applied to transformed Q. One of the most popular transformations is

$$Q'_{i,j} = \ln(Q_{i,j}) \qquad (6)$$

Q´ is then treated in the same way as Q. Theoretically runoff cannot be Gaussian distributed as runoff has a definite lower limit of zero, but inflow has no fixed lower limit. Anyhow, there exist cases where the normal distribution can be a good approximation even to the runoff process, a necessary but not sufficient condition is that runoff very seldom approaches zero. The normal distribution is used mainly in modelling runoff over long time units, e.g. yearly runoff. Besides eq (6) there exist a number of other transformations applied in order to obtain the normal distribution, e.g. the cube-root transformation.

An essential feature of these models is that they are to a greater or lesser extent centered around the weekly distributions of inflow. This means that if the purpose is to generate synthetic sequences which preserve some parameters from the historic series of the weekly distributions, these models are well-suited. It must be remembered however, that if a transformation is applied, what will be preserved are parameters of the transformed inflow. This is why some hydrologists prefer estimating the parameters of the weekly distributions directly from original data and not from the transformed data. However, in most cases the inflow itself has a rather skewed distribution and the transformation applied at least partly removes the skewness (as this is a purpose of the transformation). This means that estimates from the transformed process are probably more stable than those from the inflow itself, a fact that supports working with transformed data.

In the models described above the pattern of persistence is governed by ρ or B.

When modelling short term runoff another problem is that the process is asymmetric in time with sharp rising parts and slow recessions. Processes based upon the normal distribution cannot take this recession phenomena into account (Weiss 1975). This asymmetry is important in short term modelling but may be ignored when modelling yearly and possibly even monthly runoff.

The model in eq (3) is linear in the inflow itself or in transformed inflow, and another matter of discussion is whether linear models are adequate for describing inflow. However linear Gaussian models are easy to use, the theory is well developed in all aspects and well documented, and such models are rather easily extended to multisite series of runoff (series consisting of runoff at several stations simultaneously).

The general theory of linear stochastic processes is treated in for example Yaglom (1962) (except parameter estimation) and Box-Jenkins (1970). For applications in hydrology some publications will be mentioned in the following, but apart from these general treatments of linear stochastic processes are found in for example Yevjefich (1972b), Kisiel (1969) and Hansen (1971). It is the opinion of the author that if one wants to use linear Gaussian models, desirable properties of the real runoff process are:

a) $u_j \gg s_j$ which means that the runoff very seldom approaches its lower limit zero, this implication holds for all realistic probability distributions.

b) The point at which the runoff is considered should be situated so far downstream in the river (or below a large enough lake) that the streamflow process does not exhibit sharp peaks as a regular feature.

But even if condition b) is not satisfied the models may be successfully applied to runoff averaged over longer time intervals (a year, maybe a month).

If the discharge point is situated below a large lake we will probably need a model in which current runoff is explicitly dependent on several previous values, but this presents no great problems using linear Gaussian models, one possibility is eq (3) with m rather high.

Thomas and Fiering (1962) were probably the first to use the model in eq (2) in generating synthetic sequences of runoff. They applied the model to monthly streamflow. This model implies Gaussian distributed runoff unless a non-linear transformation of Y is made. Another way to take departure from normality into account was applied in Thomas and Fiering (1963). They replaced E in eq (2) by ϵ defined by

$$\epsilon_{i,j} = (2/\gamma_\epsilon) \cdot (1 + (\gamma_\epsilon \cdot E_{i,j}/6) - (\gamma_\epsilon^2/36))^3 - (2/\gamma_\epsilon) \qquad (7)$$

where γ_ϵ skewness of ϵ.

$$\gamma_\epsilon = ((1-\hat{\rho}_Y^3)/(1-\hat{\rho}_Y^2))^{1.5} \cdot \hat{\gamma}_Y \qquad (8)$$

$\hat{\gamma}_Y$ estimated skewness of Y.

$\hat{\rho}_Y$ estimated lag-one autocorrelation of Y.

Y generated according to this scheme will be approximately gamma distributed. In Matalas (1967) methods of using models similar to eq (2) but with non-normal residuals E is further discussed and extensions to multistation models are also treated. Gottschalk (1975) applied models of the same kind as eq (3) to monthly river runoff at various sites in Sweden. He transformed the runoff values to the unit normal distribution and applied a general method of extending the model to multisite processes. Also contained in his report are (among others) a treatment of various transformations to the unit normal distribution, various probability distributions of monthly runoff, methods of parameter estimation, and regional variations of statistical parameters of the runoff process.

Another problem in generating synthetic sequences of inflow by random methods (Monte-Carlo methods) is the long term properties. For example if one generates a series of weekly inflows and aggregates the series to a sequence of yearly totals, there is no guarantee that important characteristics of the yearly inflows is reasonably preserved, not even if the model works well with respect to the weekly distributions. If the correlation between different years is non-vanishing, this is often the most critical parameter; yearly sequences generated by aggregation often have less persistence than the historic series. The "Hurst phenomenon", another long-term characteristic will be briefly discussed at the end of this chapter.

The inability of for example monthly models to give reasonable traces of annual runoff gave rise to the "Disaggregation" model by Valencia and Schaake (1973) which was refined by Mejia and Rouselle (1976). The model accepts annual flows generated by any model (including the historic series) and disaggregates the streamflow into shorter time intervals. Here the disaggregation of yearly inflows into quarterly ones is explicitly shown, disaggregating yearly values into for example monthly ones is done in a similar way.

$$Y = A \cdot X + B \cdot Z + C \cdot E \qquad (9)$$

where all elements in Y, X and Z have been adjusted to zero mean. X is a scalar denoting the annual inflow, Y is a 4 x 1 matrix of quarterly inflows, Z is a p x 1 matrix designed to take into account the correlation between quarterly values of last year with values of present year, and E is a 4 x 1 matrix of independent residuals with zero mean and unit standard deviation. A is a 4 x 1 matrix, B is a 4 x p matrix, and C is a 4 x 4 matrix. We also want to obtain.

$$X = D \cdot Y \tag{10}$$

where $D = (1 \ , \ 1 \ , \ 1 \ , \ 1 \ )$

If the parameters in the model are estimated according to the method described below, it can be shown (see Mejia and Rouselle 1976) that eq (10) will hold exactly. Moreover, the correlations between X and Y and between different elements of Y (the quarterly inflows) are also preserved. By choosing the vector Z properly we can also preserve correlations between quarterly values present year and quarterly values last year. If we only want to preserve the correlations between quarter 4 year t-1 and the values of year t, p equals 1 and Z is put equal to the forth element in Y for year t-1.

$$A = (S_{YX} - S_{YZ} \cdot S_{ZZ}^{-1} \cdot S_{ZX}) \cdot (S_{XX} - S_{XZ} \cdot S_{ZZ}^{-1} \cdot S_{ZX})^{-1} \tag{11}$$

$$B = (S_{YZ} - A \cdot S_{XZ}) \cdot S_{ZZ}^{-1} \tag{12}$$

$$C \cdot C^T = S_{YY} - A \cdot S_{XX} \cdot A^T - A \cdot S_{XZ} \cdot B^T - B \cdot S_{ZX} \cdot A^T - B \cdot S_{ZZ} \cdot B^T \tag{13}$$

where in general S_{uv} means $E\{u \cdot v^T\}$

$(C \cdot C^T)$ is a symmetrical matrix which can be solved for C by e.g. the principal component technique (see e.g. Anderson 1958). In hydrology the annual series often can be considered stationary, but runoff over time intervals shorter than a year cannot owing to the seasonality. Some stochastic models are easily adopted to phenomena possessing seasonality, but there exist models often used in modelling long-term persistence which can be applied to stationary processes (or to be exact processes where the stationarity assumption is a reasonable approximation) only. One way to solve this problem is to choose any good model of annual runoff and then use the disaggregation method to obtain seasonal values. Synthetic sequences generated in this way preserve important characteristics of the autocorrelation structure both with regard to long-term and to short-term properties. But somehow generation of seasonal values by disaggregation seems improper, because in nature the opposite occurs; the "generating"

starts with seasonal values and it is the yearly values that are obtained by aggregation.

Moreover, if the residuals (the elements of E) are normally distributed the difference between rising and recession parts of the real process raises the same problems as models like eq (3).

If we had a perfect short term model, the long term series obtained by aggregating the synthetic series generated with the model into for example yearly values would preserve also the important characteristics (including persistence) of the annual series. But such a good all round stochastic model is unknown to the author, and if it is ever constructed he fears that it will be very complicated, maybe extremely complicated in its structure of persistence; current inflow will perhaps explicitly depend on a long sequence of past values.

3.3 MODELS BASED ON PHYSICAL OR PROBABALISTIC MECHANISMS

Yakowitz (1973) and Weiss (1973, 1977) applied models that can be considered links between weather-runoff models and stochastic runoff models. Their models consist of deterministic recession parts which govern the process in times of no input (rain), but the input processes are stochastic. This seems like rainfall-runoff models, but the runoff is n o t generated from precipitation. Instead the input process is modelled from the streamflow series only.

Weiss (1973, 1977) applied "Shot-Noise Models" for the generation of daily runoff from some English streams. The process (X(t)) is defined by

$$X(t) = \sum_{m=N(-\infty)}^{N(t)} y_m \cdot \exp(-b \cdot (t - \tau_m)) \qquad (14)$$

y	random variable having the probability distribution $F(y) = (1/\Theta) \cdot \exp(-y/\Theta)$.
y_m	corresponds to τ_m.
N(t)	a Poisson process with rate ν.
$\ldots \tau_{-1}, \tau_0, \tau_1 \ldots$	times of events of the process N(t).
b	decay rate (a constant)
X(t)	streamflow.

The physical interpretation of the process (mainly from the papers of Weiss) is:

The events at the random times τ_m can be thought of as the starts of rainstorms, y_m associated with τ_m corresponds to the volume of rainfall in the rainstorm starting at τ_m.

$$y_m \cdot \exp(-b \cdot (t - \tau_m))$$

is the transfer function from rainstorm to runoff and the decay rate b depends on catchment characteristics. It can be shown that eq (14) can be written:

$$X(t+s) = (\exp(-b \cdot s)) \cdot X(t) + \epsilon_s(t+s) \qquad (15)$$

which is a first order autoregressive process, but it is not a Gaussian process because $\epsilon_s(t+s)$ is non-negative with a skewed distribution and with a non-zero probability of being exactly zero. Seasonality can be taken into account by using different parameter values during different parts of the year, Weiss (1973, 1977) estimated the parameters separately for each calendar month. In estimating ν, Θ and b Weiss applied a method which ensures preservation of (daily) mean, standard deviation and lag-one (lag = 24 hours) autocorrelation. The alternative approach to estimate b directly from the recessions and ν by observing times of peaks in the data was also tried. But the model fitted in this way did not preserve the daily means, standard deviations and lag-one autocorrelation, so Weiss preferred the former approach.

This simple shot noise model did not perform satisfactory on the English streams chosen for study. The synthetic monthly runoff obtained by aggregation of daily data had much smaller standard deviations and lag-one serial correlation coefficients than the historical series. This is a common problem encountered in aggregation, but perhaps more serious drawbacks of the simple shot noise model were that in generated data the recessions approached zero too fast and too many rises and recessions appeared. This discrepancy was ascribed to the base flow component, and the model was generalized to a sum of two simple shot noise processes, one fast and the other slow. These new models performed better. For further details see the papers by Weiss, which also include extensions to multisite models.

Yakowitz (1973) applied his model to Rillito, a stream which during a large proportion of the days has zero runoff. He did not use the same deterministic part as Weiss, and input was defined to have occurred when the runoff exceeded that predicted by the deterministic recession part. Even if this input certainly is closely related to precipitation, it is not identically equal to it. The details of the model are not given here. See the paper by Yakowitz.

The Rec-Ris model developed in this paper (Chapter 4) has similarities with both the shot-noise models and with the model of Yakowitz,

Markov Chains and some of its extensions have been applied to hydrology by several researchers. For its use in modelling inflows and runoff see e.g. Lloyd (1967, 1977) and Jackson (1975 a,b,c). Here the inflows must be grouped into classes, e.g. n classes. The state of the process is defined by the class to which the inflow belongs, and the process is described by matrices like:

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & \\ \cdots & \cdots & \cdots & \\ \cdots & \cdots & \cdots & \\ p_{n1} & p_{n2} & \cdots & \end{pmatrix} \qquad (16)$$

$$\sum_{j=1}^{n} p_{ij} = 1 \qquad (i = 1,2,\ldots,n) \qquad (17)$$

$$p_{ij} = \text{Prob}(S_{t+1} = j \mid S_t = i)$$

S_t state of the process during time step t.

This is a finite-state Markov Chain, chains with an infinite number of states will not be dealt with here. Here both time and state are discrete but otherwise these processes are rather general; no assumptions on linearity, stationarity, symmetric distribution etc. are needed. By letting p_{ij} vary cyclically with t one can get processes with seasonality, and (at least in theory) it is even possible to get a process with annual non-stationarities in mechanism by letting p_{ij} depend on the year.

Above it is assumed that

$$\text{Prob}\,(S_{t+1} = j \mid S_t = i \wedge S_{t-1} = i_{-1} \wedge \ldots \wedge S_{t-L} = i_{-L}) =$$

$$\text{Prob}\,(T_{t+1} = j \mid S_t = i)$$

This is called the Markov postulate. By changing the definition of the state this assumption is not necessary. If defining the state not only by the class of present inflow but by both the class of present and the class of last inflow, one also obtains a matrix similar to eq (16).

A general treatment of Markov Chains is beyond the scope of this report, see e.g. Cox-Miller (1965), Feller (1957). There exist methods of stochastic optimization where we are satisfied by just having the Markov matrices in eq (16) and do not need any generations at all (Willén 1978). But often the Markov Chain is used in synthetic generation, and then the output of this is just a sequence of states, S_t. In some cases one is satisfied with this series of quantisized inflow, but as the classes may be rather wide, we often want more detailed traces of inflow.

Jackson (1975 a, b, c) used three methods to achieve this. In the first model the flow value is selected entirely at random from the appropriate distribution independent of previous state. This means that the only influence of previous flow on the present one is on its state. In model 2 the value of flow (after its class has been chosen by the Markov Chain) is determined by

$$(Q_{t+1} - u_j)/s_j = \rho \cdot (Q_t - u_i)/s_i + \sqrt{1 - \rho^2} \cdot \epsilon_{t+1} \qquad (18)$$

ϵ	independent identically distributed random numbers with zero mean and unit standard deviation.
i(j)	class of current (next) flow.
u_i (u_j)	expected value of flows in class i (j).
s_i (s_j)	standard deviation of flows in class i (j).
ρ	the correlation which is a constant independent of i and j.
Q_t	flow in time step t.

In model 3 ρ is dependent on the classes of current and next flows. Another possibility is to replace the rows in eq (16) of $(p_{i1}, p_{i2}, \ldots\ldots, p_{in})$ with conditional distributions

$$F(X \mid i) = \text{Prob}(x_{t+1} \leq X \mid x_t \text{ belongs to class i}) \qquad (19)$$

This approach which was suggested by prof Erik Eriksson (personal communication) is applied to seasonal inflows in chapter 5 of this report.

3.4 HURST PHENOMENON AND ANNUAL MODELS

Although the emphasis in this report is on short-term modelling, a brief discussion of some problems of long-term modelling is included. The range (defined below) is a parameter of long-term series which is very much discussed.

$X_1, X_2, \ldots\ldots, X_n$ a sequence of annual inflows into a reservoir over n years.

$$D_n = \sum_{i=1}^{n} X_i \qquad (20)$$

D_n represents the total amount of inflow to the reservoir during n years. As a mean value of the amount of water released from the reservoir over a long time horizon we should have

$$(1/n) \cdot \sum_{i=1}^{n} X_i \quad \text{each year.}$$

Supposing that this amount of water is released each year, we have after k years

$$D_k^* = \sum_{i=1}^{k} X_i - (k/n) \cdot \sum_{i=1}^{n} X_i = D_k - k \cdot D_n/n \qquad (21)$$

where D_k^* represents the excess (may be negative) relative to the amount released up to year k. The range (R_n) is defined by

$$R_n = P_n - Q_n \qquad (22)$$

where

$$P_n = \max\ (D_k^*) \qquad k = 1, 2, \ldots, n$$

$$Q_n = \min\ (D_k^*) \qquad k = 1, 2, \ldots, n$$

Hurst (1951) found R_n to vary with n according to

$$(R_n/S) \sim n^h \tag{23}$$

S sample standard deviation of the series of length n.
h a constant, sometimes called the "Hurst coefficient"

with mean of h equal to 0.73. He studied many natural annual time series comprising runoff, precipitation, tree rings etc. The series varied in length between 30 to 2000 years. The problem is that annual models based on

$$y_i = \sum_{k=1}^{m} B_k \cdot Y_{i-k} + \sigma \cdot E_1 \tag{24}$$

asymptotically (for large n) leads to h = 0.5. This equation corresponds to eq (3) with seasonality removed and defines an autoregressive process of order m.

B_k autoregressive coefficients.

σ a constant.

E_i a sequence of independent random numbers with the unit normal distribution.

Y_i denotes inflow transformed to zero mean and unit standard deviation.

The controvercy concerning h (often named the "Hurst coefficient") led to development of new models for annual runoff, models designed to take this phenomena into account. One important generalization of eq (24) is

$$Y_i = \sum_{k=1}^{m} B_k \cdot Y_{i-k} + \sum_{k=0}^{p} A_k \cdot E_{i-k} \tag{25}$$

which defines the Arma (m, p) process. A and B are parameters to be estimated.

In eq (25) the first term on the right side corresponds to the polynomial

$$\varphi(b) = 1 - \sum_{k=1}^{m} B_k \cdot b^k \tag{26}$$

I can be shown that if eq (25) is to define a stationary process, all roots of $\varphi(b) = 0$ must lie outside the unit circle (see for example O´Conell 1974) and here we limit ourselves to the stationary versions of eq (25).

Asymptotically, that is for very long sequences, series generated by eq (25) will have h = 0.5 in eq (23), but even an Arma (1,1) process may have a long transient with h confirming to Hursts law. And maybe even the natural hydrologic series ultimately approaches the h = 0.5 law, but we do not know if this is the case or not until long enough records of hydrologic quantities are available.

O´Conell (1974) applied the Arma (1,1) process to annual streamflow series from different parts of the world. For an extensive discussion on long term modelling in general and Arma processes in hydrology the reader is referred to the report by O´Conell, which also contains a physical basis for the Arma (1,1) model for annual runoff postulated by Moss (1972).

The autoregressive models have finite memory in the sense that current inflow is an explicit function of a finite portion of the history of the process only. Another approach to modelling of long term persistence (including Hurst´s law) is to consider models with infinite memory. Such a model is the Fractional Gaussian Noise model, which adequately accounts for the Hurst law (Mandelbrot-Wallis 1968, 1969a,b,c,d,e). Over time periods of sufficient length the climate will change and as the climatic cycles are irregular this is an argument for using models with infinite memory, or at least models with very long memory.

Fractional Gaussian Noise ($B_h(t)$) may be defined by

$$B_h(t)-B_h(0)=(1/\sqrt{h+0.5}).\left[\int_{-\infty}^{0}\left((t-u)^{h-0.5}-(-u)^{h-0.5}\right)dB(u)+\int_{0}^{t}(t-u)^{h-0.5}\,dB(u)\right] \qquad (27)$$

$$0<h<1$$

where dB(u) is an infinitesimal increment of Brownian motion. In eq (27) h is the Hurst coefficient, which is thus an explicit parameter of the model. A drawback of Fractional Gaussian Noise models is the excessive computor time required in generating synthetic series, but approximations exist which are better in this aspect (see for ex. Mandelbrot 1971).

The Fractional Gaussian Noise and the Broken Line (BL) (to be defined below) are processes that cannot be applied to seasonal phenomena easily.

Rodriguez et al (1972) and Mejia et al (1972) presented the "Broken Line" process as a model of the Hurst law and long term persistence in general.

To cite from Mejia et al (1972):

"The simple broken line process results from the linear interpolation between equally spaced independent Gaussian random variables and the random displacement of the starting point of the series in order to make it stationary". As an example see Fig 2 below.

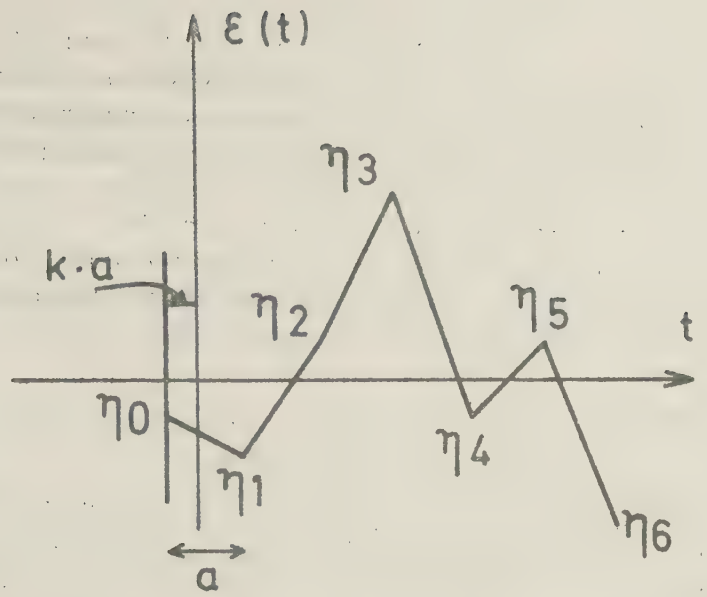

Fig 2 Illustration of a simple Broken Line process.

The definition of a simple BL-process is given by

$$\epsilon(t-k\cdot a) = \sum_{n=0}^{\infty} (\eta_n + (\eta_{n+1} - \eta_n)\cdot(t-n\cdot a)/a)\cdot I\left[na,\ (n+1)a\right](t) \qquad (28)$$

$\eta_0, \eta_1 \cdots, \eta_n \cdots$ independent and identically distributed random variables with zero mean.

k random variable uniformly distributed between zero and one.

a time distance among the η .

I indicator function defined by

$I[na, (n+1)a](t)=1$ $na \leq t \leq (n+1)a$

$I[na, (n+1)a](t)=0$ otherwise

It can be shown (Meija et al 1972) that

$$\begin{aligned} \rho(\tau) &= 1-3\cdot(\tau/a)^2\cdot(2-(\tau/a))/4 && 0 \leq \tau \leq a \\ \rho(\tau) &= (2-(\tau/a))^3/4 && a \leq \tau \leq 2a \\ \rho(\tau) &= 0 && \tau > 2a \end{aligned} \qquad (29)$$

where $\rho(\tau)$ is the autocorrelation at lag η .

The general BL process is defined as the sum of a number (N) of simple broken line processes ($\epsilon_i(t)$).

$$XBL(t) = \sum_{i=1}^{N} \epsilon_i(t) \qquad (30)$$

The author considers the BL-process a rather artificial model that is

further away from physical relaties than most other models in use.

Another approach to Hurst law is annual processes. Klemes (1974) and Potter (1975-1976) consider shifts in the central tendency as a possible explanation of the Hurst phenomenon, and their research are supported by stochastic models that preserve the phenomenon for very long time periods.

Their explanation is reasonable because even if one often tests annual historic series against unstationarity before modelling them by stochastic processes, the type II errors are serious. A typical test work according to:

Hypothesis H_0: The series i stationary.

Hypothesis H_1: The series is not stationary.

A small number (α) is chosen as the significance level of the test, which means that the probability of rejecting H_0 when it is in fact true equals α. This is called type I error, but there also exists the possibility of a type II error, which occurs if H_0 is accepted when it is in fact false. α is usually chosen rather small (for example 0.10, 0.05), the historic series are often not long and in fact the hypothesis H_1 includes many kinds of non-stationarities. This explains why type II errors are serious problems. Indeed, even if the result of the test applied is that H_0 is accepted, we could probably formulate some kinds of hypothesis on non-stationarity that also would pass some tests.

The author thinks that the merit of statistical tests is that they may reject bad hypothesis, but in order to accept a hypothesis statistical tests are not powerful. They should be supplemented by physical or phenomenological arguments if this is possible.

4 MODELLING OF PENTADE RUNOFF AT YTTERHOLMEN

4.1 INTRODUCTION

The model applied consists of two parts, a snow-melt model which determines runoff during pentades 25-31, and "rec-Ris" which is used during the rest of the year. See Fig 3 for location map.

4.2 REC-RIS

The idea of this model is to take into account the quite different mechanisms involved during the recession and rising parts of runoff. When runoff increases it is due to external factors, i.e. the weather in case of unregulated flow. But the decreasing parts of the runoff series are governed mainly by internal factors, viz. catchment characteristics. This is a simplification because in case of a small decrease in runoff the reason may be a nonzero rainfall which is too small to balance the recession mechanisms in the real catchment, and likewise if runoff is increasing the decay of flow caused by the recession mechanisms may be considerable even if they are more than balanced by a large input (rain). The model is applied to pentade (five-day mean) runoff at Ytterholmen (river Råneå) in north Sweden. In Fig 11 the historic series (1924-67) is shown and in Tables 2 and 3 some parameters from this and generated series are shown. The drainage area is 1040 km^2 and the area is occupied by 2.3% lakes. The runoff usually is maximum during spring owing to snowmelt, it decreases during the summer, sometimes increases during autumn due to increased precipitation and decreased evaporation, and reaches minimum during winter because of snow accumulation.

Let us describe Rec-Ris.

$$Q_{t+1} = \begin{cases} Q_t + D_{t+1} & Q_{t+1} > Q_t \\ R_{t+1} \cdot Q_t & Q_{t+1} \leq Q_t \end{cases} \qquad (31)$$

Q_t runoff during time step t (pentades).
D_t positive random numbers.
R_t random numbers between Rmin and one.
Rmin the lowest value R is allowed to take.

In this chapter the unit of runoff is 100 l/s.

First one has to decide whether the runoff is increasing or not. This is governed by p_{01} and p_{11} defined by

$$p_{01} = \text{Prob}\ (Q_{t+1} > Q_t \mid Q_t \leq Q_{t-1})$$

$$p_{11} = \text{Prob}\ (Q_{t+1} > Q_t \mid Q_t > Q_{t-1})$$

where in general $p_{11} > p_{01}$.

Of course these probabilities vary with the time of the year because of the weather. At first we attempted not to distinguish between p_{01} and p_{11} but as they are considerably different we had to do so.

After having determined if next runoff is increasing or not we generated D_{t+1} or R_{t+1} according to their probability distributions. In the early stages of developing this model the idea was to have the probability distributions of D as function of season only and that of R as function of Q only. The arguments are that D is closely related to rainfall which shows seasonal variation and R is related to the state of the catchment, especially the wetness of it and Q can be considered an index on this wetness. The basis of having R as a random variable is that Q is not a perfect index on the state of the catchment, recession in our model is defined to occur also in times of nonzero precipitation if it is too small to fully compensate for the decay mechanisms of flow, and recession is also influenced by evapotranspiration which is considered to be a stochastic process.

However it soon became apparent that R was also season-dependent and D also dependent on Q.

The influence of season on R is probably explained by evapotranspiration which is maximum during summer, decreases during autumn, is probably negligible during winter and increases during spring etc. The influence of evapotranspiration on R is not limited to the evaporation from lakes and the river, because evapotranspiration is a drying process, and this drying is (at least to some extent) transferred from the surface layers of the soil to the groundwater. We generally have R decreasing with Q. This is because if Q is high a higher part of it derives from overland flow than if Q is low, when it is derived mainly from groundwater storage, and groundwater reservoirs are rather slow.

Whether D should increase or decrease with Q is not so easy to postulate, at least the author had some difficulties in doing it. One argument is that high Q means that the decay is rather fast and this means that a certain amount of precipitation is used in balancing this decay, but if Q is low the decrease produced by the recession is rather low and most of precipitation is used in increasing runoff. But data analysis showed positive correlation between D_{t+1} and Q_t!

How to explain this? One possibility suggested by professor Erik Eriksson (personal communication) is that in case of high Q_t most infiltration areas

have been transformed into outflow areas because high runoff indicate that the catchment is rather wet. Another though related explanation is that the dryer the state of catchment, the greater amount of precipitation is used to wet the soil. Also the persistence in weather may contribute to the positive correlation, anyhow it is probably the reason for $p_{11} > p_{01}$.

As a model of D the following relation is used

$$\ln(D_{t+1}) = A + B \cdot \ln(Q_t) + ED_{t+1} \cdot SE \qquad (32)$$

A, B, SE seasonal parameters.
ED independent random numbers with unit normal distribution.

This model is reasonable because D_t fits the log-normal distribution well for most parts of the year, and to have ln(D) as a linear relation to Q_t certainly would imply too fast rising parts for high Q_t. Even this "log-log" model needed a few corrections in order to prevent explosive behavior, these corrections are described in the section on testing. In order to have R constrained between Rmin and unity and negatively correlated to Q_t we applied

$$R_{t+1} = Rmin + (1-Rmin) \cdot ((RM-Rmin)/(1-Rmin))^{AR \cdot Q_t \cdot E_{t+1}} \qquad (33)$$

where

RM mean of R(=0.8879).
AR seasonal parameter.
E random number with the exponential distribution

$$F(E) = 1 - \exp(-E) \qquad (34)$$

It should be noticed that in eq (33) a high value of E corresponds to a low value of R, and E=0 gives R=1.

4.3 SNOW-MELT MODEL

During spring, runoff is fed by both precipitation and snow-melt. In principle we can consider p_{01}, p_{11} and D related to snow-melt as well as rainfall precipitation, but snow-melt also means a depletion of the snow magazine, a magazine which is (except in extreme cases) not refilled until next winter. This depletion means, that use of Rec-Ris is questionable especially during the late stages of snow-melt. In an ordinary winter the snow-cover lasts to the end of pentade 26 (to 10/5), but even if we have bare ground from pentade 27 on, it may be reasonable to extend the snow-melt model beyond this pentade, because of the inertia of the process. Moreover, in some years the snow-melt is late. The start of snow-cover usually is 25/10. Our stochastic snow-melt model works in two main steps:

i) A prediction of "total runoff" during pentades 24-35 is made.
ii) Based on this prediction we fix Q for each of the pentades 25-31.

In i) total runoff is defined as

$$QV = \sum_{p=24}^{35} Q_p \tag{35}$$

and in predicting QV we use the simple linear regression

$$\hat{QV} = AH + BH \cdot QH \tag{36}$$

where QH is defined as

$$QH = \sum_{p=15}^{22} Q_p \tag{37}$$

The correlation between QV and QH is positive (0.6618), and similar results were obtained for the inflows to Korsvattnet and Stora Mjölkvattnet (see next chapter). These results have surprised some hydrologists, who argue that high QH $\Longrightarrow$ large snow-melt during the winter $\Longrightarrow$ rather small amounts of snow remain at the time of ordinary snow-melt. But when snow-melt occurs during winter, it is generally caused by westerly cyclones, and these cyclones usually lead not only to snow-melt but also to snow-precipitation. In addition, not all westerly cyclones produce snow-melt but almost all of them produce precipitation. This means that large snow-melt during winter is usually accompanied with even larger snow-precipitation. Killingtvweit et al (1976) also used winter runoff as a predictor of snow-melt runoff.

An alternative explanation of the positive correlation between winter runoff and spring runoff is suggested by professor Erik Eriksson (personal communication). His idea is that a large snow magazine will force water out from the groundwater magazine by its pressure force, i.e. winter runoff should be an index of the snow cover.

A third explanation of the positive correlation obtained is, that high runoff during winter is due to rather full groundwater magazines, which owing to its inertia will continue to deliver plenty of water to runoff for some time ahead. At Ytterholmen this mechanism cannot be the main explanation however, because of the usually large fractions of spring runoff which are derived by snow-melt, the same holds for Korsvattnet and Mjölkvattnet. (When stating that large fractions of spring runoff are derived from snow-melt we do not discuss whether melt-water from snow enters the streams directly or if it enters the soil and forces groundwater out into the streams.) In south Sweden we would probably obtain negative correlation between winter runoff and "ordinary" snow-melt runoff.

Several periods for defining QV and QH were tried, and also linear regression of QV on ln(QH) was tried, but equations (35), (36), (37) gave the highest correlation. Q is determined by

$$\ln(Q_{p+1}) = \ln(a_p) + b_p \cdot TK_p + c_p \cdot \ln(Q_p) + s_{\epsilon(p+1)} \cdot \ln(\epsilon_{p+1}) \tag{38}$$

or

$$Q_{p+1}=a_p \cdot \exp(b_p \cdot TK_p) \cdot Q_p^{c_p} \cdot \epsilon_{p+1}^{s_{\epsilon(p+1)}} \qquad (39)$$

a_p, b_p, c_p, $s_{\epsilon(p+1)}$ coefficients used in determining Q for pentade p+1 (p=24,24,...,30).

$\ln(\epsilon_p)$ independent random numbers with the unit normal distribution.

That is $s_{\epsilon(p+1)} \cdot \ln(\epsilon_{p+1})$ is the residuals for $\ln(Q_{p+1})$ in the linear regression model eq (38).

$$TK_p = \sum_{j=23}^{p} Q_p / \widehat{QV} \qquad (40)$$

The parameters are determined separately for each pentade. We expected and obtained $b < 0$ and $c > 0$. TK is the fraction of total predicted spring-runoff that has already passed away before the next time step.

Because $\widehat{QV}$ is just a prediction of QV we sometimes obtain TK > 1. The positive values of c is a product of the inertia in the real process.

For pentades 31,32,... Q is determined by Rec-Ris. As $\widehat{QV}$ obtained in eq (36) is just a prediction of QV we have no guarantee that the correlation (=0.66) between QV and QH is preserved in generated sequences. The results of Monte-Carlo experiments gave a somewhat lower correlation, see section 7 of this chapter.

While Rec-Ris is considered a phenomenological model, our snow-melt model is based mainly on regression.

4.4 GENERATION SCHEME

A) pentades 25-31.
1) Generate a unit normally distributed random number ϵN.
2) Use eq (38) where $\ln \epsilon = \epsilon N$.
3) Compute the new value of TK.
4) Return to 1) if next pentade is not 31, else go to B.

B) Rest of the year (Rec-Ris model).
1) Generate ϵ, a random number rectangularly distributed between zero and one.
2) Define $p_1 = \begin{cases} p_{01} & \text{if } Q_t \le Q_{t-1} \\ p_{11} & \text{else} \end{cases}$
3) If $(\epsilon \geq (1-p_1))$ Go to 9.
4) As $\epsilon < (1-p_1)$ we have recession.
5) EPS = $\epsilon/(1-p_1)$.
6) $E_{t+1} = -\ln(1-Eps)$.

7) Compute R_{t+1} from eq (33).

8) $Q_{t+1} = R_{t+1} \cdot Q_t$, then go to 14.

9) We have rising runoff.

10) $EPS = (\epsilon - (1-p_1))/p_1$.

11) Transform EPS to the unit normally distributed ED_{t+1}.

12) Compute D_{t+1} from eq (32).

13) $Q_{t+1} = Q_t + D_t$.

14) If runoff for pentade 22 is the latest generated value, compute QH and $\hat{QV}$ by equations (37) and (36). If next time step is pentade 25, go to step 1 of A, else return to step 1 of B).

4.5 PARAMETER ESTIMATION

First the parameters are estimated separately for each pentade, or in cases with few events in single pentades, for each group of pentades. The yearly variation of parameters obtained are smoothed by applying harmonic regression.

$$V_p = AV_o + \sum_{i=1}^{NH} \left[AV_i \cdot \cos(2 \cdot \pi \cdot i \cdot p/73) + BV_i \cdot \sin(2 \cdot \pi \cdot i \cdot p/73) \right] \qquad (41)$$

V_p value of parameter V in pentade p.

AV_o, AV, BV harmonic coefficients for parameter V.

NH number of harmonics used.

This approach was applied to p_{01}, p_{11}, A, B, SE and AR. In Table 1 the harmonic coefficients are shown for each parameter. Turning to the probability distribution of E in eq (33), the exponential distribution $F(E)=1-\exp(-\lambda \cdot E)$ was applied, but both mean and standard deviation of E computed from the data were near one (1.0141 and 1.01113 respectively). For the exponential distribution the mean equals the standard deviation, and the estimate of λ obtained by methods of moments is $\lambda = 1/\text{mean}$, so we put $\lambda = 1$ and obtained the distribution in eq (34).

AH and BH in eq (36) are determined by the ordinary linear regression method. a_p, b_p, c_p and s_ϵ are determined by multiple linear regression (eq (38)). In Table 1 the values of AH, BH, a, b, s and s_ϵ are shown.

The estimation methods used are thus simple, in later development of the Rec-Ris and snow-melt models maybe use will be made of more sophisticated estimation methods.

Rmin was (rather arbitrarily) put equal to 0,8 · (the smallest value of R in the historic series).

Table 1. Parameters of our model of pentade runoff at Ytterholmen

	p_{01}		p_{11}		A	
i	AV_i	BV_i	AV_i	BV_i	AV_i	BV_i
1	-0.1275	-0.0039	0.0007	0.0734	-1.0836	-0.2280
2	-0.0948	-0.0187	-0.1513	0.0294	-0.1281	-1.0739
3	0.0928	-0.0027	0.0779	-0.0162	0.5152	-0.0984
4	-0.0035	0.0368	-0.0166	0.0847	-0.4112	-0.0409
5	-0.0196	-0.0392	0.0016	-0.0411	-0.0468	-0.4694
6	-	-	0.0146	-0.0007		
7	-0.0048	0.0072	-0.0275	-0.0053		
8	0.0075	-0.0101	0.0118	-0.0102		
9	0.0035	0.0072	0.0032	0.0356		
AV_o	0.1738		0.7036		-2.5717	

	B		SE		AR	
i	AV_i	BV_i	AV_i	BV_i	AV_i	BV_i
1	0.5512	0.3276	-0.2163	-0.1523	-0.0270	-0.0441
2	0.0251	0.4859	0.0236	-0.1499	-0.0236	0.0218
3	-0.0704	0.1269	0.0161	-0.0127	-0.0256	-0.0204
4	0.1116	0.1665			0.0024	0.0011
5	-0.0478	0.1511			0.0089	-0.0027
6					-0.0021	0.0118
AV_o	1.5282		1.1348		0.1233	

AH = 187.36 BH = 12.06 Rmin = 0.2861 RM = 0.8879

p	a_p	b_p	c_p	$s_{\epsilon(p+1)}$
24	0.2813	-12.7864	1.4680	0.5042
25	0.9638	- 5.6370	1.1160	0.5838
26	1.0278	- 3.7169	1.0616	0.4924
27	1.4919	- 1.9942	0.8280	0.5016
28	1.6224	- 1.5093	0.7525	0.3487
29	1.1624	- 1.0738	0.8103	0.3356
30	0.0675	- 0.4122	0.9749	0.3062

4.6 PRINCIPLES OF TESTING

Two main approaches has been used in testing the model.

A) When generating a synthetic series one generates a random number in each time step (pentade). The sequence of "random" numbers required to give the historical series of runoff is computed. In most cases (see generation scheme) the random number generated is rectangularly distributed between zero and one. And if the "random" number does not have this distribution we can transform it into the desired distribution by a method similar to that applied in eq (5). This transformation to the rectangularly distribution between zero and one is applied when according to the generation scheme we should generate a random number which does not have this distribution. The argument of applying this transformation is a desire to have uniform testing conditions for all times of the year.

This gives a sequence, u_t, which if the model is good should appear as a sequence of independent rectangularly distributed random numbers $0 \leq u \leq 1$.

When computing the sequence u_t the program starts with runoff pentade 1 year 1924 as known, and as increasing flow is very unusual during this time of the year, it was assumed that the last step was a recession. Here the first "random" number corresponds to pentade 2 year 1924 and the last one, u for pentade 72 during 1967, corresponds to the flow pentade 73 this year. The Kolmogorov-Smirnov test (see for example Yevjevich 1972a) was used for each of the 73 pentade distributions of u in testing if it had the desired rectangular distribution.

The "run-Test" and "reverse arrangements" were also applied. In the type of run-test used here, a run is defined by a sequence of value below or above the median, i.e., as we have 0.5 as median, the sequence.

0.1 0.2 0.6 0.8 0.9 0.1 0.6

has 4 runs. See for example Bendat-Piersol (1971) or Hansen (1971). In the "reverse arrangements" one counts how many times one has $u_j > u_i$ when $j < i$. See for instance Otnes - Enochsson (1972) or Hansen (1971). When testing against a monotonous trend the reverse arrangements - tests is the better one, whereas the run-test is to be preferred when testing independence.

B) Monte-Carlo generated series were compared with the historical (1924-1967) record in several ways.

4.7 RESULTS

In generated sequences occasionally extremely high runoff was generated during winter and autumn. These extreme flows were the result of the rather high p_{11} values during autumn and high values of B during winter. In some cases this led to a long sequence of rising steps, which together with eq (32) sometimes raised the flow beyond realistic values. This forced us to modify the model. The first idea was that in case of high Q the positive correlation between D_{t+1} and Q_t would disappear or at least decrease. But no such tendencies could be found in the historic series, instead there was an d e c r e a s e (but very small) in correlation if the highest values of Q_t and the corresponding D:s were d e l e t e d from analysis during autumn, and during winter the record contained no cases with increasing runoff when Q was greater than 5 units. A reasonable explanation is, that if Q_t is very high, the probability of Q_{t+1} being even higher should be smaller than if Q_t is smaller. The definitions of p_{01} and p_{11} were changed according to

$$\begin{aligned} p_{11} &= \text{Prob}(Q_{t+1} > Q_t \mid Q_t > Q_{t-1} \wedge Q_t < Qlim) \\ p_{01} &= \text{Prob}(Q_{t+1} > Q_t \mid Q_t < Q_{t-1} \vee Q_t \geq Qlim) \end{aligned} \qquad (42)$$

and the corresponding changes were made in the generating scheme. Qlim was put equal to 20 units, a value that was chosen rather arbitrarily but after testing some other values. We put B equal to zero in eq (32) for t corresponding to p = 1,2,....,24 if $Q_t > 5$ units, because we did not want very sharp rising parts above this value, as the historical record contained none. Another discrepancy between the historical and generated series was the extremely high runoff values during snow-melt that were obtained several times in each 50-year series. In order to correct for this, part A) of the generating scheme was modified, so that if $Q > 200$ units was generated, a new generation was made and the new value was accepted even if it was > 200 units.

Three synthetic series of length 50 years were generated. The first of them and the historical record 1924-67 are displayed in Fig:s 12 and 11 respectively. In Tables 2-5 some parameters of the historical and generated series are compared and in Fig:s 4 and 5 the yearly variations of the means and standard deviations of synthetic series no. 1 and the historical one are shown.

The result of this Monte-Carlo experiment is that especially the synthetic series no. 1 looks good.

In the other synthetic series the standard deviations are considered too high during some pentades of the autumn. This is caused by an almost explosive behaviour during autumn which (even if it is very unusual) occurs sometimes in spite of the modifications made (eq (42)), i.e. the upper extremes of flow during autumn are not stable. The correlation between QH and QV are rather low for the third series, but the correlation coefficient is a parameter subject to intense sampling variations.

Anyhow, this correlation was smaller than in the historic record in all our generated series, so probably we must refine our snow-melt model.

The explosive behaviour of runoff that still sometimes occurs during autumn in generated series should be removed in future versions of Rec-Ris. One possibility is to split p_{11} into different values depending on e.g. how many time steps in succession there were increases.

Considering the fact that the parameters compared are not used in model building and parameter estimation (except maybe for QV), the model performs rather good.

Turning to the sequence u of test A, the results were not promising in all aspects. The 73 pentade distributions were tested by the Kolmogorov-Smirnov test.

Results: significance level 0.20 gives 24 rejections (pentades 4, 7, 9, 10, 11, 16, 18, 19, 20, 23, 24, 31, 32, 38, 40, 48, 52, 58, 59, 65, 66, 68, 71, 72)
significance level 0.10 gives 16 rejections (pentades 7, 10, 11, 18, 19, 20, 23, 31, 32, 40, 48, 58, 59, 65, 68, 72).

The number of rejections is too high.

Table 2. Pentade means of the historical series (1924-67) and 3 generated 50-year series of runoff at Ytterholmen. Unit: 100 l/s.

Pentade	Historical records	Synthethic series 1	Synthetic series 2	Synthetic series 3
1	4.15	4.00	4.41	3.72
2	3.94	3.83	4.09	3.58
3	3.74	3.69	3.90	3.50
4	3.53	3.54	3.72	3.32
5	3.36	3.37	3.54	3.23
6	3.21	3.27	3.36	3.12
7	3.09	3.18	3.20	3.01
8	2.99	3.12	3.10	2.91
9	2.90	3.03	3.02	2.85
10	2.81	2.98	2.91	2.79
11	2.73	2.89	2.84	2.74
12	2.68	2.80	2.78	2.67
13	2.64	2.70	2.72	2.62
14	2.60	2.60	2.66	2.56
15	2.53	2.52	2.57	2.49
16	2.47	2.48	2.49	2.42
17	2.41	2.46	2.45	2.35
18	2.36	2.47	2.41	2.32
19	2.37	2.47	2.41	2.31
20	2.46	2.57	2.46	2.43
21	2.71	2.75	2.80	2.64
22	3.53	3.33	3.28	2.98
23	5.53	4.12	3.86	4.51
24	9.27	5.68	6.13	5.93
25	18.84	14.26	15.53	13.56
26	37.86	38.83	36.84	29.09
27	62.34	67.96	61.99	63.71
28	70.65	73.67	69.18	65.85
29	62.00	65.76	63.71	61.23
30	49.26	50.68	52.28	51.34
31	37.26	38.82	40.77	40.41
32	28.64	28.84	31.63	30.39
33	24.27	24.55	29.14	24.54
34	21.26	17.47	21.77	20.14
35	16.97	14.47	16.46	13.96
36	14.00	10.89	13.69	11.38
37	11.99	9.25	12.90	8.90
38	10.65	8.32	11.99	8.46
39	9.74	9.87	11.43	9.07
40	8.37	10.12	9.68	7.58

Table 2 (cont) Pentade means of the historical series (1924-67) and 3 generated 50-year series of runoff at Ytterholmen. Unit: 100 l/s.

Pentade	Historical records	Synthetic series 1	Synthetic series 2	Synthetic series 3
41	7.78	9.92	9.94	7.38
42	8.65	9.42	13.68	7.07
43	8.56	8.31	9.75	7.41
44	9.17	7.12	10.07	7.08
45	9.20	9.34	11.04	7.90
46	9.13	9.22	8.76	7.74
47	9.41	9.58	8.91	7.72
48	9.88	11.13	8.65	8.09
49	9.98	9.63	8.98	7.42
50	10.49	8.75	10.02	9.07
51	11.15	9.19	16.37	10.10
52	11.11	11.03	23.21	12.96
53	12.69	12.00	14.72	14.63
54	14.54	11.51	12.73	13.45
55	13.47	12.27	10.67	18.51
56	14.20	14.36	19.43	24.39
57	15.67	14.68	21.17	19.78
58	16.29	15.93	14.91	18.10
59	13.79	14.32	14.39	29.55
60	11.72	13.76	15.32	16.72
61	10.88	12.57	13.69	15.41
62	10.54	10.58	10.28	10.83
63	10.96	9.81	8.69	9.00
64	10.29	8.64	8.45	8.34
65	8.95	7.89	7.24	7.47
66	7.41	6.57	6.27	6.79
67	6.54	5.77	5.45	5.94
68	6.06	5.31	5.17	5.63
69	5.95	5.06	5.03	5.08
70	5.95	5.28	4.78	4.72
71	5.20	4.82	4.90	4.41
72	4.71	4.47	4.71	4.15
73	4.37	4.20	4.63	3.97

Table 3. Standard deviations of pentade runoff at Ytterholmen of the historical series (1924-67) and 3 generated 50-year series. Unit: 100 l/s

Pentade	Historical records	Synthetic series 1	Synthetic series 2	Synthetic series 3
1	2.34	2.05	2.80	1.80
2	2.12	1.92	2.29	1.64
3	1.89	1.86	2.10	1.61
4	1.68	1.78	1.91	1.41
5	1.55	1.66	1.72	1.37
6	1.41	1.62	1.57	1.26
7	1.28	1.60	1.42	1.19
8	1.17	1.58	1.34	1.11
9	1.05	1.52	1.29	1.07
10	0.97	1.51	1.20	1.04
11	0.85	1.30	1.11	1.02
12	0.79	1.16	1.05	0.96
13	0.75	1.01	1.02	0.93
14	0.73	0.95	0.99	0.89
15	0.68	0.96	0.88	0.87
16	0.65	1.01	0.85	0.84
17	0.64	1.03	0.83	0.81
18	0.64	1.14	0.82	0.78
19	0.68	1.16	0.85	0.73
20	0.78	1.24	0.98	0.94
21	1.30	1.36	1.28	1.09
22	4.07	1.69	1.73	1.32
23	10.70	2.09	2.26	5.06
24	17.92	2.97	4.36	5.70
25	24.81	11.65	16.47	12.30
26	34.18	32.44	38.25	26.56
27	43.15	42.95	45.62	47.73
28	38.07	38.78	36.49	42.00
29	31.78	32.56	33.42	29.96
30	26.78	30.18	23.88	21.70
31	24.99	29.16	21.64	21.17
32	22.71	27.15	17.41	15.16
33	25.09	21.64	21.57	16.54
34	21.76	10.03	13.76	13.76
35	14.46	8.20	8.27	7.10
36	9.07	4.81	7.08	5.69
37	6.97	4.26	8.67	3.13
38	5.87	3.57	9.02	3.86
39	6.25	7.79	9.21	8.40
40	5.08	8.17	6.40	4.65

Table 3 (cont) Standard deviations of pentade runoff at Ytterholmen of the historical series (1924-67) and 3 generated 50-year series. Unit: 100 l/s.

Pentade	Historical records	Synthetic series 1	Synthetic series 2	Synthetic series 3
41	5.73	9.19	11.19	5.27
42	9.69	8.62	35.55	5.83
43	8.72	9.94	12.48	6.88
44	10.40	5.71	13.12	4.64
45	8.53	8.60	17.53	7.15
46	8.30	7.24	8.37	5.23
47	8.49	8.35	7.29	4.95
48	9.29	14.81	6.35	5.91
49	10.45	7.28	6.97	4.29
50	12.76	6.15	10.09	8.38
51	12.04	6.87	49.84	19.51
52	9.20	13.61	93.72	16.07
53	11.41	13.12	30.76	18.99
54	14.47	9.72	20.77	13.69
55	11.83	9.98	13.94	24.03
56	15.04	15.51	53.04	44.89
57	19.53	12.19	67.71	27.35
58	17.92	16.25	22.39	23.06
59	11.61	13.12	14.02	74.97
60	7.79	14.03	17.76	23.52
61	7.37	10.56	19.57	24.22
62	7.19	8.00	8.50	8.88
63	11.00	6.75	6.82	6.48
64	9.85	5.76	7.37	6.59
65	6.90	6.12	5.28	5.34
66	4.71	4.06	4.24	5.10
67	3.89	3.46	3.14	3.95
68	3.59	3.04	2.94	4.04
69	4.48	2.98	2.80	3.48
70	6.24	5.15	2.66	2.81
71	3.89	3.59	3.68	2.52
72	3.02	2.98	3.05	2.24
73	2.58	2.37	3.07	2.10

Table 4 Some parameters of spring-runoff (QV) at Ytterholmen of historic (1924-67) and 3 generated 50-year series. Unit: 100 l/s. ρ (QV, QH) is the correlation between QV and QH.

	Historical series	Synthetic series 1	Synthetic series 2	Synthetic series 3
Mean	438.6	441.0	445.4	420.1
Standard deviation	142.0	159.9	138.5	116.0
γ_1	0.56	2.01	0.60	-0.06
γ_2	3.96	11.19	3.32	3.61
ρ (QV, QH)	0.66	0.63	0.52	0.33

Table 5 Some parameters of the yearly mean runoff at Ytterholmen from historic (1924-67) and 3 synthetic 50-year series. Unit: 100 l/s.

	Historical series	Synthetic series 1	Synthetic series 2	Synthetic series 3
Mean	12.17	11.95	12.71	12.05
Standard deviation	2.93	3.36	6.06	3.97
γ_1	0.49	0.25	3.56	1.61
γ_2	2.94	3.20	20.3	6.11

γ_1 and γ_2 are the skewness and kurtosis respectively. For a variable with n observations γ_1 and γ_2 are computed by (Yevjevich 1972a).

$$\gamma_1 = \frac{\text{third central moment}}{(\text{second central moment})^{3/2}} \cdot \frac{n^2}{n \cdot (n-1)} \tag{43}$$

$$\gamma_2 = \frac{\text{fourth central moment}}{(\text{second central moment})^{2}} \cdot \frac{n^3}{n \cdot (n-1) \cdot (n-2)} \tag{44}$$

Fig 3 Location map for Ytterholmen

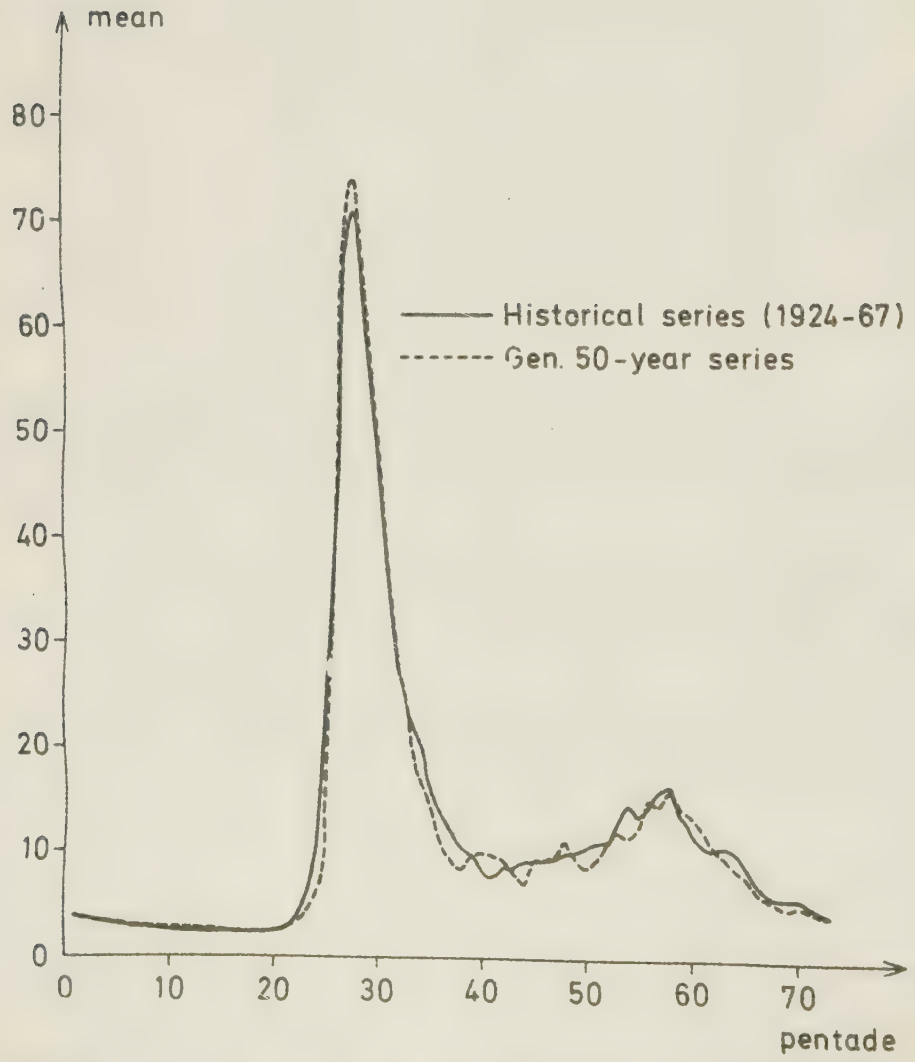

Fig 4 Means of pentade runoff for the historical (1924-67) record and synthetic series no 1. Unit: 100 l/s.

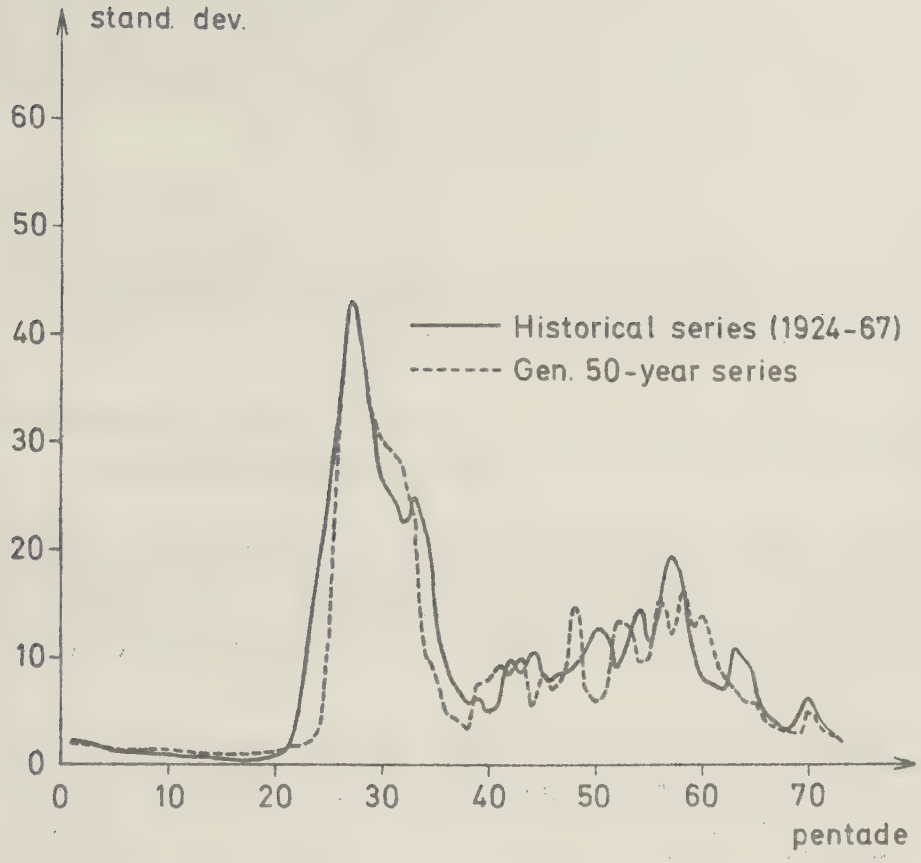

Fig 5 Standard deviations of pentade runoff for the historical (1924-67) record and synthetic series no 1. Unit: 100 l/s.

The run-test and numbers of reverse arrangements were applied to both the whole sequence u_t and to a "mean year" of u formed by taking the mean values of u for each of the 73 pentades. This new sequence is called $\bar{u}$. The $\bar{u}_t$:s should of course be independent and without trends. The results are shown in Table 6 below.

Table 6. Results of run-test and rev arr for Ytterholmen

		Expected value	Standard deviation	Obtained
u	Number of runs	1606.5	28.3	1010
	No of rev arr	2576827	30332.7	2554580
$\bar{u}$	Number of runs	37.5	4.2	28
	No of rev arr	1314	105.0	1649

For the sequence u the amount of data is rather high (3111), which means that the distributions of number of runs and number of reverse arrangements can be considered Gaussian. The number of runs is really too low, but the number of reverse arrangements is reasonable. Considering $\bar{u}$

the approximation with Gaussian distributions cannot be made, but the number of reverse arrangements is rather high. The number of runs is low but may be reasonable; the hypothesis of independent $\bar{u}$ cannot be rejected on the 5% significance level. For further details we refer to the references on run-test and reverse arrangements mentioned above.

The studies on $\bar{u}$ are considered to be tests on the modelling of seasonality.

The mean and standard deviation for the rectangularly distribution applied are 0.5 and 0.296 respectively. The results from u were 0.491 and 0.293 respectively.

The results of test A indicate that the model is probably inadequate in some aspects and it is the intent of the author to refine it.

Test A applied here is conceptually similar to testing of the residuals in autoregressive models, in both cases there exists a one to one relationship between the next flow value and a random number. But considering Shot-Noise models and the model of Yakowitz (1973) such tests are not easily applied.

It is possible that the relative sensitivity of u due to Q is greater than that of Q due to u, but this is not known. If this is the case it may partly explain the fair results of test A. In the historic record we often have R=1 exactly, but this is certainly due to the fact that Q is given only by 2 decimals and due to measurement errors. R=1 is obtained almost only during winter and for low values of Q, which means that in reality we probably have R slightly less than one.

In order to account for this a "randomization" procedure was applied when computing u in the cases where $Q_{t+1}=Q_t$.

First

$$RL = \text{min of } R = (Q_{t+1} - 0.005)/(Q_t + 0.005) \qquad (45)$$

was defined, then the value of u corresponding to RL was computed and finally u was chosen at random between this value and zero (which corresponds to R=1). It is not known if this method of "randomization" is the best one.

The number of parameters in our model is not an exact quantity. It depends on what definitions we use in model building, but let us make an "estimation". In Table 1 there are a total of 110 parameters and in our record (1924-67) we have 3212 values of runoff. This gives about 29 data values/parameter and it is considered satisfactory.

5 STOCHASTIC MODELS OF WEEKLY INFLOWS TO KORSVATTNET AND STORA MJÖLKVATTNET. MAINLY BASED ON CONDITIONAL DISTRIBUTIONS.

5.1 INTRODUCTION

The historical series of weekly inflows to the Korsvattnet and Stora Mjölkvattnet reservoirs are displayed in Fig 15. Some parameters of the records (1945-73 for Korsvattnet and 1945-75 for Stora Mjölkvattnet) are shown in Figs 7-10 and in Tables 12-15. The catchment areas are 85 and 147 km^2 respectively, and the dams are located in the mountains of Jämtland in central Sweden. The altitudes are about 740 and 540 m a s l respectively and the latitude between 63^o and 64^o north, see location map in Fig 6. The inflows are expressed in VE, which is the unit used throughout this chapter.

1 VE = the amount of water corresponding to 1 m^3/s during one week.

The inflow series show similar features as the runoff record from Ytterholmen, but at least one important difference exists. Rising parts during winter are very seldom at Ytterholmen but here it is more usual, though recession is the ordinary process during winter even here. The reason of this difference in winter flow is probably that these parts of western Jämtland are more open to the influence of westerly cyclones than the north of Sweden, and these cyclones sometimes lead to snow-melt even during mid-winter.

The first step in developing these models was to apply the Markov Chain (eq (16)) to weekly inflows to Korsvattnet. As the purpose is to model weekly inflows, we would like to have one matrix of type (eq (16)) corresponding to each week, i.e. 52 matrixes. But it would require an enormous amount of data to obtain reasonable estimates of all these; for Korsvattnet we only have 29 years (for Stora Mjölkvattnet 31 years) of weekly data, and if we choose six classes, we would only have about 5 "events" in each class. This forces us to compromise; we keep the same matrix for several weeks during the year and change matrices only a few times a year. We can use, say, one matrix for winter, one for spring and early summer and one for late summer and autumn, i.e. three periods.

How many classes and how many periods to use is a matter of discussion, it is a compromise between the desire not to have too many parameters in the model and the wish to model the process in a realistic way.

In the first attempt we used three Markov-matrices like eq (16) and the periods were

Weeks 1-14 and 49-52		period 1	
15-26	"spring"	2	(46)
27-48		3	

The first problem is grouping into classes including how many classes to use. Suppose one chooses to have all classes equally wide (the difference between max and min inflow the same in all classes). Then, however, the classes corresponding to the highest inflows will get rather few data. It was therefore decided to arrange the grouping in such a way that all classes would contain roughly the same number of events and the number of classes chosen was six.

But this leads to another problem if the model is to be used in generating synthetic flows; the class-limits will be different in different periods and (during a generation) when passing from one period to another the choice of class is not trivial. In order to solve this problem, but also in order to specify Q more exactly than by just its class, the Markov Chain was modified according to the method mentioned in chapter 3 (eq (19)). This means that we replaced the rows in eq (16) of $p_{i1}, p_{12}, \ldots, p_{in}$ with conditional distributions

$$F(X \mid i) = \text{Prob}(x_{t+1} \leq X \mid x_t \text{ belongs to class } i) \qquad (47)$$

Here X denotes untransformed inflow in VE.

It should be mentioned that there exists optimization models in water resources theory where one does not use generated sequences but works directly with the discrete time-discrete state Markov Chain, see references in Willén (1978). As we had 6 classes, we had 6 conditional distributions for each period, i.e. a total of 18.

5.2 PARAMETER ESTIMATION

For every period and every class i we select (from the historical series) all X_{t+1} such that X_t belongs to class i. There are, e.g., N such X_{t+1}:s, and these N values are sorted according to:

$$X^1 \leq X^2 \leq \ldots\ldots \leq X^{N-1} \leq X^N \qquad (48)$$

(here the superscript is an index).

Then the plotting formula

$$P(X^m) = m/N+1) \qquad m = 1, 2, \ldots, N \qquad (49)$$

(Yevjevich 1972a) is used. This gives us a discrete approximation of the conditional distribution function $F(X \mid i)$. For $F(X \mid i)$ the following analytical

expression was used

$$F(X \mid i) = \begin{cases} 1-a_1 \cdot \exp(-a_2 \cdot (X-a_3)^{IE1}) - (1-a_1) \cdot \exp(-a_4 \cdot (X-a_3)^{IE2}) \\ \qquad \text{if } X > a_3 \\ 0 \qquad \text{else} \end{cases} \tag{50}$$

a_1, a_2, a_3, and a_4 are parameters which of course depend on the class and period. Moreover we have the constraints

$$\begin{aligned} &0 \leq a_1 \leq 1 \\ &a_2 \geq 0 \\ &0 \leq a_3 \leq X^1 \\ &a_4 \geq 0 \end{aligned} \tag{51}$$

and we chose the alternatives

$$(IE1,\ IE2) = (1,\ 2),,\ (1,\ 3),,\ (2,\ 3),\ (2,\ 4) \tag{52}$$

The parameter vector $\underline{a}(=a_1, a_2, a_3, a_4)$ was determined for each of the four combinations of IE1 and IE2. It was always possible to get a reasonable result with at least one of the combinations. In hydrology the Maximum-Likelihood method (ML-method) is often used in parameter estimation. However, the author does not consider the method proper in the present case. In the ML-method one asks: What value should be given to the parameter-vector $\underline{a}$ in order to make the realisation of the process (i.e. here the sequence in eq (48)) as likely as possible? But this question can be answered only, if we know the kind of distribution that really governs the process, i.e. what kind of distribution that has produced the realisation. This is not known here. (Are there ever cases where this knowledge is at hand?)

Instead, we argue: Let us find an approximation, that closely follows the points $P(X^m)$ in eq (49), and we may try an expression like eq (50) without stating that the real distribution governing the process could be written in this way. That is, we only assume that this realisation from the process closely follows an expression of this kind, if we use suitable parameter values. Even if the real distribution governing the process is unknown, it is reasonable to assume, that the real distribution should follow the discrete approximation $P(X^m)$ produced by the realisation rather close. When determining the parameters a least squares method is used.

$$\text{Min } D = \sum_{m=1}^{N} \left(F(X^m \mid i) - P(X^m)\right)^2 \tag{53}$$

It is not known if the least squares method is the best one, but it seems reasonable and since (as far as the author knows) it is not known what criterion is best, it is used. The method has the merit (see Fig 13) that the points corresponding to F almost one are given low weights in the for-

mula (53). These points are the most uncertain in the sense that another realisation of the same process may produce quite different X^m especially for the highest values of m. The ML-method does not generally have this merit.

However we have paid a certain price for obtaining estimates which are rather insensitive to extreme points. Our estimation method can be considered a method that optimizes $\underline{a}$ with respect to F and not to X. This means that the upper tail of the estimated distribution is maybe even more uncertain with respect to X than the distribution obtained by for example a usual method of least squares described below. Here $\underline{a}$ is determined by

$$\text{Min } DX = \sum_{m=1}^{N} (X(F^m)-X^m)^2 \tag{54}$$

which in the corresponding sense means optimizing with respect to X. Here $X(F^m)$ means the value of X which gives $F=P(X^m)$. The weakness of this method is the weight of extreme points. The choise of method is not trivial. The purpose of the stochastic models in this case is generation of synthetic series of weekly inflows by random numbers. And these synthetic sequences are then used in optimization of releases for hydro-electricity production, see Willén (1978). Although we work on weekly time units and are interested in weekly inflows, the detailed distributions of extremes for single weeks are mostly not critical in our applications. More important is the distributions of sums of inflows over some weeks.

When minimizing in eq (53), the Simplex-method of nonlinear optimization (Nelder-Mead 1965, according to Parkinson, J.M. and Hutchinson, D. 1971 loc.cit) has been used, after transforming our optimization problem according to:

$$\begin{aligned} a_1 &= \sin^2 b_1 \\ a_2 &= b_2^2 \\ a_3 &= X^1 \cdot \sin^2 b_3 \\ a_4 &= b_4^2 \end{aligned} \tag{55}$$

This well-known transformation method leads to an unconstrained optimization problem.

5.3 GENERATION OF SYNTHETIC INFLOWS

The procedure is as follows:

1. Choose a suitable starting value for week 52 year 0 (choose for example the median value for the historical values of week 52).
2. Go through the following steps with start at week 1 year 1 in the generated series.

A) Determine the period to which the previous inflow belongs.

B) Determine its class (=i).

C) Generate a random number ϵ with rectangular distribution $0 \leq \epsilon \leq 1$.

D) Solve the equation $F(X \mid i) = \epsilon$ for X. This gives the generated inflow. This is then used as "past inflow" when generating the inflow of next week.

As the uncertainty in F is greatest for F near one, upper limits are put on X for every class and period. However, only the very upper part of F has been truncated, never below F = 0.95 and mostly above. The place of truncation depends on the shape of F compared with its discrete approximation $P(X^m)$. If a random number higher than the truncation point is generated, the flow corresponding to the truncation point is accepted as the generated inflow.

In the historical ceries the sum of inflows during period 2 is always rather high, i.e. there is always "spring-inflow". But in the generated series the spring-flow was almost absent more than once every ten years in a generated sequence of 50 years. The model was also tried with slightly different definitions of period 2, but the results were roughly the same. Then the model was modified for the spring-period, where a special model to be described in the next section was applied.

5.4 SNOW-MELT MODEL

Some definitions:

QV — sum of inflows during spring-period (here weeks 15-26).

Q_V — inflow during week V.

R_V — rest of QV before start of week V.

$$QPR_V = Q_V/R_V$$

It was found that QV could be considered approximately normally distributed. For weeks 16, 17,....,25 distributions were fitted to the discrete distributions of QPR generated by our historical series, one distribution for each week. For weeks 16, 17, 18 and 19 the exponential distribution gave good fit and for weeks 20, 21,....,25 the Gaussian distribution worked well. When estimating the parameters the method of moments (see for example Yevjevich 1972a) was applied.

Q_{15} is generated by looking at Q_{14} and choosing the appropriate conditional distribution function as described in section 5.3 Q_{26} is simply R_{26}, and then Q_{26} is used to generated Q_{27} in the ordinary non-spring way etc.

The lag-one autocorrelation of QPR for weeks 21, 22, 23, 24 and 25 was taken into account using one single correlation coefficient for all these weeks.

That is QPR_{20} is generated without considering QPR_{19} but for 21 26:

$$(QPR_V - QM_V)/S_V = AR \cdot (QPR_{V-1} - QM_{V-1})/S_{V-1} + \sqrt{1-AR^2} \cdot EV \quad (56)$$

where

AR autocorrelation coefficient.

E_V a sequence of independent random numbers with unit normal distribution.

QM_V mean of QPR_V.

S_V standard deviation of QPR_V.

The approach to fix the sum of spring inflows before it is distributed over the weeks has a physical reality, because a very high proportion of this sum is derived from snow-melt. As an approximation it can be stated that nature first sets the sum of these inflows, and then (mainly by temperature) distributes it in time. Another model of snow-melt was applied in chapter 4, where the sum of spring-flow is predicted and not fixed. This approach will probably also be applied to Korsvattnet and Stora Mjölkvattnet, but it is not contained in the present paper.

5.5 RESULTS OF TESTING

Test B and (after appropriate modifications) test A, as described in chapter 4, were applied to the model of Korsvattnet. The periods were slightly redefined according to

Period 1		weeks 1-14 and 47-52
2	(as before)	15-26
3		26-46

First let us describe the results of test A.

The run-test and no. of reverse arrangements were applied to the whole sequence u_t (week 1 year 1945 - week 50 1973). As the number of data is 1506, the distributions of both no. of runs and no. of reverse arrangements can be considered Gaussian. No. of runs and of rev. arr. differed from their expected values; 1.39 below and 0.9 above respectively (expressed in standard deviation units).

The sequence $\bar{u}_t$ (t=1,2,....,52) was formed and tested by the run-test and no. of rev. arr..

Results: no. of runs 24 which is within the 0.10 significance level and
no. of rev. arr. 685 which is within the 0.10 significance level.

The 52 weekly distributions of u were tested for having the desired rectangular distribution by the Kolmogorov-Smirnov test.

Fig 6 Location map for Korsvattnet and Stora Mjölkvattnet.

Results: significance level 0.20 gives 12 rejections (week 4, 13, 14, 26, 32, 33, 34, 39, 43, 45, 47, 48)
significance level 0.10 gives 5 rejections (weeks 34, 39, 43, 45, 47).

The outputs of test A must be considered good.

Turning to test B, a 250 year series of weekly inflows was generated with our model and compared with the historical record. The unconditional weekly distributions of generated and historical series were plotted using eq (49) to get a rough idea of how the model worked for each of the 52 weeks.

The Kolmogorov-Smirnov test was then applied to each of the 52 weeks by comparing the two unconditional distributions. When making this test it was assumed, as an approximation, that the generated series had given us the same weekly distributions as the population (of this special stochastic model). We investigated, if the weekly distributions produced by our historical series differed significantly from those of the "population".

Results: significance level 0.20 gave 14 rejections (weeks 14, 24, 27, 30, 34, 37, 38, 39, 40, 41, 44, 45, 46, 47)
significance level 0.10 gave 11 rejections (weeks 24, 27, 37, 38, 39, 40, 41, 44, 45, 46, 47).

Test A did not make the model appear bad, but what about the outputs of test B? They are difficult to interpret, but do not appear very good for the autumn. We state the hypothesis that our historical 29,year series might well be a realisation of our model. Of course different 29-year realisations will give different weekly distributions, this concerns all 52 weeks. If all the 52 weekly distributions produced by a 29-year realisation were independent, it would be rather much to have 11 rejections on the 0.10 level and to have 9 weeks of the autumn rejected on the 0.20 level. But these weekly distributions are not independent. During a generation the distribution of Q_{V+1} is dependent on Q_V, so if among the 29 years the inflows of a certain week V happened to be in general rather high, then also the inflows of week V+1 are likely to be rather high. When looking at the same test on u_t, the result was more promising, and here all the 52 weeks should be considered independent. This means, that the hypothesis that the rather high inflows during some weeks in the autumn (compared with inflows generated by the model) is entirely a result of chance cannot easily be rejected by the results of our tests.

But even if the statistical tests applied above cannot reject the model, other arguments are against it. We should not be satisfied just by having a model such, that the historical series might well be a realisation of it, a characteristic which is also studied by test A. We want a model whose realisations in general are reasonable candidates of future real series. In this case there are physical arguments of inflow being higher during autumn than during late summer. The evapotranspiration is generally higher during summer than during autumn owing to the temperature effect, and precipitation usually increases from July to October. At least the temperature effect should persist in future series because of astronomical realities.

In order to take this into account, we revised our model to include three periods plus the spring. The same kind of model was also applied to Stora Mjölkvattnet.

5.6 NEW VERSIONS OF MODELS FOR KORSVATTNET AND STORA MJÖLKVATTNET

The periods are redefined according to

Period	1		weeks 1-14 and 44-52
	2	"spring"	15-26
	3		26-33
	4		34-43

The parameter a_3 in eq (46) is dropped by replacing it with zero, but otherwise the same kind of conditional distributions and method of estimation are used for periods 1, 3 and 4. The conditional distributions for Korsvattnet are displayed in Fig 13 and the corresponding parameters are contained in Tables 7 and 8.

Also, the spring models are modified. We still start by fixing QV, but the positive correlation between winter inflow (QH) and QV is taken into account. QV is now generated by

$$(QV-\overline{QV})/S_{QV} = \rho \cdot (QH-\overline{QH})/S_{QH} + \sqrt{1-\rho^2} \cdot E \qquad (57)$$

ρ correlation coefficient between QH and QV.

$\overline{QV}$, $\overline{QH}$ mean of QV and QH respectively.

S_{QV}, S_{QH} standard deviation of QV and QH respectively.

E independent random number with unit normal distribution.

QV is defined as earlier in this chapter, and

$$QH = \sum_{V=47}^{52} Q_V(\text{last year}) + \sum_{V=1}^{14} Q_V(\text{present year}) \qquad (58)$$

for Korsvattnet and

$$QH = \ln\left(\sum_{V=51}^{52} Q_V(\text{last year}) + \sum_{V=1}^{14} Q_V(\text{present year})\right) \qquad (59)$$

for Stora Mjölkvattnet.

The choise of periods defining QH and of lin-lin or lin-log relations was made by the criterion of highest possible correlation. The correlations obtained are 0.6832 for Korsvattnet and 0.7275 for Stora Mjölkvattnet.

The physical reasons of positive ρ was described in chapter 4.

Also the distribution of QV over the weeks was slightly changed. Now we consider QPR either normal (N) or lognormal (LN) distributed, and the autocorrelation of QPR is taken into account for all the weeks. The distributions of QPR and the autocorrelations are shown in Tables 9 and 10.

Table 7 Parameters of the conditional distributions for Korsvattnet.

Period	1	a_1	a_2	a_4	IE1	IE2	
Class	1	0.851	3.89	0.320	1	2	$Q < 0.3$
	2	0.808	1.45	3.43	1	3	$0.3 \leq Q < 0.6$
	3	0.445	1.34	1.23	1	2	$0.6 \leq Q < 0.9$
	4	0.380	0.921	0.541	1	3	$0.9 \leq Q < 1.3$
	5	0.419	0.745	0.227	1	3	$1.3 \leq Q < 1.9$
	6	0.362	0.362	0.132	1	3	$Q \geq 1.9$
Period	3						
Class	1	0.858	0.840	0.0271	1	3	$Q < 1.4$
	2	0.839	0.439	0.288	1	3	$1.4 \leq Q < 2.3$
	3	0.560	0.0685	0.100	2	4	$2.3 \leq Q < 3.1$
	4	0.632	0.0536	0.0240	2	4	$3.1 \leq Q < 4.6$
	5	0.256	0.130	0.0119	1	3	$4.6 \leq Q < 7.5$
	6	0.992	0.0168	0.530	2	3	$Q \geq 7.5$
Period	4						
Class	1	0.808	0.449	45.7	1	2	$Q < 1.2$
	2	0.879	0.212	0.00370	2	3	$1.2 \leq Q < 2.0$
	3	0.691	0.231	0.00472	2	3	$2.0 \leq Q < 2.8$
	4	0.308	0.218	0.0228	1	3	$2.8 \leq Q < 3.8$
	5	1.0	0.0420	(0.7)	2	(3)	$3.8 \leq Q < 5.4$
	6	0.546	0.0259	0.0199	2	3	$Q > 5.4$

Notice: The numbers in parenthesis have no meaning as $a_1 = 1$ exactly.

Table 8 Parameters of the conditional distributions for Mjölkvattnet. Unit: VE

Period 1		a_1	a_2	a_4	IE1	IE2	
Class	1	0.187	0.553	0.983	1	2	$Q < 1.0$
	2	0.443	0.239	0.402	2	4	$1.0 \leqslant Q < 1.4$
	3	0.508	0.223	0.196	2	4	$1.4 \leqslant Q < 1.8$
	4	0.614	0.156	0.0899	2	4	$1.8 \leqslant Q < 2.4$
	5	0.141	0.0962	0.0623	1	3	$2.4 \leqslant Q < 3.3$
	6	0.125	0.00695	0.0228	2	3	$Q \geq 3.3$
Period 3							
Class	1	0.609	0.404	0.0942	1	3	$Q < 1.9$
	2	0.416	0.0233	0.218	2	3	$1.9 \leqslant Q < 3.5$
	3	0.273	0.109	0.0115	1	3	$3.5 \leqslant Q < 5.1$
	4	0.509	0.0141	0.0356	2	3	$5.1 \leqslant Q < 7.7$
	5	0.694	0.0153	0.00100	2	4	$7.7 \leqslant Q < 11.6$
	6	0.696	0.00514	0.0000230	2	4	$Q \geq 11.6$
Period 4							
Class	1	0.560	0.605	0.0420	1	2	$Q < 1.9$
	2	0.199	0.101	0.0669	1	2	$1.9 \leqslant Q < 3.2$
	3	0.523	0.0155	0.0615	2	3	$3.2 \leqslant Q < 4.5$
	4	0.578	0.0120	0.0110	2	3	$4.5 \leqslant Q < 6.2$
	5	0.773	0.0208	0.000163	2	3	$6.2 \leqslant Q < 9.8$
	6	0.743	0.00970	0.00860	2	3	$Q \geq 9.8$

Table 9 Parameters of the spring-period for Korsvattnet. Unit: VE

V	15	16	17	18	19	20	21
Distr.	LN	LN	LN	LN	LN	N	N
QM	-4.860	-4.656	-4.655	-3.891	-2.923	0.123	0.1 2
S	0.8060	0.7661	0.7203	0.9662	0.8826	0.0845	0.0886
AR	-	0.4234	0.4234	0.4234	0.4234	0.4234	0.4234

V	22	23	24	25
Distr.	N	N	N	N
QM	0.249	0.322	0.406	0.564
S	0.1146	0.106	0.123	0.108
AR	0.4234	0.4234	0.4234	0.4234

$\overline{QV}$ = 94.96 S_{QV}=18.70

$\overline{QH}$ = 20.54 S_{QH}=10.47

Table 10 Parameters of the spring-period for Mjölkvattnet. Unit: VE

V	15	16	17	18	18	20	21
Distr.	LN	LN	LN	LN	LN	LN	LN
QM	-4.868	-4.648	-4.167	-3.516	-2.542	-2.14[illegible]	-1.840
S	0.6978	0.9226	0.8379	0.9461	0.7995	0.7[illegible]1	0.5656
AR		0.5804	0.5804	0.5804	0.5804	0.5804	0.5804

V	22	23	24	25
Distr.	LN	LN	LN	LN
QM	-1.472	-1.176	-0.948	-0.587
S	0.4570	0.3121	0.3075	0.1820
AR	0.3354	0.3354	0.3354	0.3354

$\overline{QV}$ = 163.30 S_{QV}=33.91

$\overline{QH}$ = 3.34 S_{QV}= 0.16

QM and S are the means and standard deviations respectively of QPR in case of Normal distribution and of ln(QPR) in case of log-normal distribution.

During generation eq (56) is appropriate modified in case of log-normally distributed QPR.

These models were subject to the same tests as the earlier versions, and the results obtained were:

MJÖLKVATTNET

Test A):

u_t mean = 0.503 and standard deviation = 0.283

The deviations from the expected values of number of runs and number of reverse arrangements were 2.1 above and 0.86 below respectively, expressed in standard deviation units. Considering the weekly distributions, we obtained

significance level 0.20 gives 7 rejections (weeks 13, 33, 35, 43, 44, 47, 48)
significance level 0.10 gives 5 rejections (weeks 13, 43, 44, 47, 48)

$\bar{u}_t$ no. of runs = 19 and no. of rev. arr. = 483

The number of runs for u_t is slightly above the 5% significance limit (considering a two-sided test) but the number of rev. arr. is quite good.

For $\bar{u}_t$ the run test cannot reject the model at the 5% level, and no. of rev. arr. is between the 2% and 5% levels.

Test B):

A 250-year series was generated and tested in the same way as described above.

Significance level 0.10 gives 11 rejections (weeks 1, 2, 14, 28, 30, 44, 48, 49, 50, 51, 52)
significance level 0.20 gives 15 rejections (weeks 1, 2, 13, 14, 15, 28, 30, 34, 41, 44, 48, 49, 50, 51, 52).

The results of the tests of weekly distributions of u and of inflow are rather good. If all 52 weeks are considered independent, we should usually have 5 rejections on the 10% level; compare test of u! Considering the weekly distributions of inflow the 52 weeks are not independent, see discussion in section 5.5.

The numbers of reverse arrangements were not quite good, but when subjecting a model to many tests, we should normally have some "bad" results.

KORSVATTNET

Test A):

u_t mean = 0.498 and standard deviation = 0.284

The deviations from the expected values of no. of runs and no. of reverse arrangements were 0.23 above and 0.58 above respectively, expressed in standard deviation units. Considering the weekly distributions we obtained:

significance level 0.10 gives 3 rejections (weeks 4, 43, 46)
significance level 0.20 gives 8 rejections (weeks 4, 12, 13, 34, 39, 43, 44, 46).

$\bar{u}_t$ no. of runs = 24 and no. of rev. arr. = 617

which are very near their expected values (27 and 663 respectively).

Test B):

The weekly distributions of a generated 250-year series were tested by the Kolmogorov-Smirnov test. Results:

significance level 0.10 gives 2 rejections (weeks 37 and 44)
significance level 0.20 gives 8 rejections (weeks 5, 19, 34, 37, 41, 44, 48, 50).

The results for Korsvattnet appear good, the result of test B may even be considered to be too "good".

In Fig 14 some of the weekly distributions of the generated 250-year series and the historical series are shown for Korsvattnet.

5.7 A BIVARIATE MODEL OF KORSVATTNET AND STORA MJÖLKVATTNET

Finally our two separate unisite models of these reservoirs were combined and extended to a bivariate model. The problem is to preserve at least some of the interrelations between the two inflow-series that exist in the historical record. One possibility would be to use bivariate conditional distribution functions of the form

$$F(X(1), X(2) \mid i(1), i(2)) =$$

$$= \text{Prob}\left((x(1)_{t+1} \leq X(1) \wedge x(2)_{t+1} \leq X(2) \;\middle|\; \begin{array}{l} x(1)_t \text{ belongs to class } i\ (1) \\ \text{and} \\ x(2)_t \text{ belongs to class } i\ (2) \end{array} \right) \qquad (60)$$

Indicies 1 and 2 denote Mjölkvattnet and Korsvattnet respectively, otherwise the symbols have the same meaning as in eq (47). But this approach has severe data requirements, because F must be estimated for each combination of i(1) and i(2).

Another possibility is to look at the relations between the two sequences of u required to give the historical series. We first transform both sequences of u into the corresponding unit normally distributed u', and then R1, the lag-zero cross correlation coefficient of u', is computed. We obtained

$$R1 = 0.476$$

which is lower than the correlation between QH for the two reservoirs. When generating a bivariate series we first generate a bivariate series of unit normally distributed random numbers ϵ', transform it into a bivariate series of rectangularly distributed random numbers ϵ $(0 \leq \epsilon \leq 1)$ and proceed in the ordinary way. Of course, inflow series generated according to this scheme and value of R1 showed too low cross correlation of QH between the two reservoirs. In order to correct for this, Monte-Carlo experiments with different values of R1 were performed, and the value of R1 accepted was R1 = 0.80.

It is also considered essential to preserve the inter-reservoir correlation of QV. The generating scheme for QV is

$$\begin{pmatrix} QVM \\ QVK \end{pmatrix} = A \cdot \begin{pmatrix} QHM \\ QHK \end{pmatrix} + \begin{pmatrix} b_{11} & 0 \\ b_{21} & b_{22} \end{pmatrix} \cdot \begin{pmatrix} E(1) \\ E(2) \end{pmatrix} \tag{61}$$

where QVM, QVK, QHM, QHK and E are all expressed in standardized units, i.e. with zero mean and unit standard deviation.

QVM (QVK)	denotes spring inflow to Mjölkvattnet (Korsvattnet).
QHK (QHM)	denotes winter inflow to Korsvattnet (natural logarithm of winter inflow to Mjölkvattnet) as defined by eqs (58, 59).
E(1) and E(2)	are independent random numbers with unit normal distribution.

By postmultiplying eq (61) with (QHM, QHK) and taking expectations we obtain

$$M_1 = A \cdot M_o \tag{62}$$

where

$$M_1 = \begin{pmatrix} \rho(QVM,\ QHM) & \rho(QVM,\ QHK) \\ \rho(QVK,\ QHM) & \rho(QVK,\ QHK) \end{pmatrix} \tag{63}$$

$$M_o = \begin{pmatrix} 1 & \rho(QHM,\ QHK) \\ \rho(QHM,\ QHK) & 1 \end{pmatrix} \tag{64}$$

where ρ (u,v) denotes correlation between u and v.
A is easily obtained from eq (62).

Postmultiplying eq (61) with (QVM, QVK) and taking expectations we obtain

$$D = A \cdot M_1^T + B \cdot B^T \tag{65}$$

where

$$D = \begin{pmatrix} 1 & \rho(QVM, QVK) \\ \rho(QVK, QVM) & 1 \end{pmatrix} \tag{66}$$

$$B = \begin{pmatrix} b_{11} & 0 \\ b_{21} & b_{22} \end{pmatrix} \tag{67}$$

Substituting A computed from eq (62) into eq (65). $B \cdot B^T$ is computed and with the special lower triangular form (eq (67)) chosen for B, it is easily obtained.

Results:

$$M_1 = \begin{pmatrix} 0.7275 & 0.6553 \\ 0.6620 & 0.6832 \end{pmatrix}$$

The values in M_1 are computed from the historical series.

Turning to ρ (QHM, QHK) the value obtained from the historical series is 0.6327. This value was used when generating a synthetic 250-year series, from which we obtained ρ (QHM, QHK) = 0.6881. This latter value was then used in eq (62) and eq (65). Results:

$$B = \begin{pmatrix} 0.652140 & 0 \\ 0.614586 & 0.292552 \end{pmatrix}$$

$$A = \begin{pmatrix} 0.525310 & 0.293831 \\ 0.364451 & 0.434421 \end{pmatrix}$$

With this parameter values new bivariate series were generated by the Monte-Carlo method.

As the two univariate series obtainable from a synthetic bivariate series by considering each reservoir separately probably have the same properties with respect to test A and the weekly distributions of Q, these tests were not repeated.

However, some parameters of the historic and generated series were

compared: some cross-correlations are shown in Table 11; weekly means and standard deviations are shown in Tables 12-15 and displayed in Figs 7-10; some parameters of yearly sums, QV and QH are shown in Tables 16 and 17, and synthetic serie 2 is displayed in Fig 16.

Table 11 Some correlations between inflows to Korsvattnet and Stora Mjölkvattnet. QH are defined by eqs (58) and (59).

	annual sums	QH	QV	NY
Historic records	0.88	0.63	0.95	29
Synthetic serie 1	0.86	0.62	0.92	30
Synthetic serie 2	0.85	0.69	0.92	30
Synthetic serie 3	0.81	0.50	0.88	30
Synthetic serie 4	0.86	0.69	0.93	250

NY length (in years) of the series.
γ_1, γ_2 see eqs (43) and (44)

Here synthetic serie 1 is the first 30 years of synthetic serie 4. Considering the fact that most of our tests are on parameters not used in model building or parameter estimation, our two unisite models and the bivariate one are considered good. Especially interesting are the rather good results for annual inflow and winter inflow, quantities obtained by aggregation. Here, as in chapter 4, let us make an estimate of the number of parameters used. Considering the unisite model of Korsvattnet we have (Table 7) 54 a:s, (Table 9) 11QM:s + 11S:s + 10AR:s + means and standard deviations of QV and QH plus ρ . This gives a total of 91 parameters, which means that we have $(29.52/91) \approx 16.6$ data values per parameter, which is less than we had for Ytterholmen, but it is considered satisfactory.

Table 12 Means of weekly inflows to Mjölkvattnet from the historic and some synthetic series. Unit: VE

Week	Historic series	Synthetic serie 1	Synthetic serie 2	Synthetic serie 3	Synthetic serie 4
1	2.59	1.9	2.0	2.4	2.1
2	2.93	1.7	2.2	1.9	2.1
3	2.04	1.4	1.7	1.6	2.1
4	1.78	1.5	1.7	1.5	1.9
5	1.90	1.7	1.8	1.7	1.8
6	1.85	1.8	1.7	2.2	2.0
7	1.60	2.5	1.6	1.6	1.9
8	1.61	2.5	2.0	3.0	2.0
9	1.56	2.1	1.8	2.2	2.2
10	2.02	3.4	2.4	2.2	2.4
11	1.56	1.7	2.1	2.3	2.1
12	1.87	2.7	2.8	1.8	2.2
13	1.58	2.9	2.4	1.7	2.2
14	1.47	3.4	3.5	1.6	2.2
15	1.62	2.3	2.6	1.9	2.[illegible]
16	2.32	2.3	2.8	2.8	2.6
17	3.47	4.5	4.4	3.9	3.7
18	6.81	6.2	9.9	6.2	6.8
19	14.20	17.6	16.1	16.1	15.8
20	18.42	17.7	17.9	20.0	18.8
21	19.82	17.6	20.2	19.1	20.4
22	23.64	21.8	25.7	23.6	25.0
23	23.12	22.4	23.5	22.6	23.1
24	19.09	19.4	20.1	18.7	19.8
25	16.90	20.9	14.2	18.0	16.6
26	14.03	16.8	12.1	13.0	13.4
27	10.28	10.6	7.6	8.6	9.1
28	7.75	7.3	5.3	7.5	6.5
29	5.44	8.1	4.8	7.0	6.0
30	5.41	6.2	5.4	7.2	5.4
31	5.07	4.4	4.6	6.0	4.9
32	4.16	6.4	4.5	5.8	4.8
33	3.98	5.0	4.6	5.1	4.5
34	2.95	4.6	3.6	4.8	4.1
35	4.11	4.8	3.8	3.8	4.7
36	5.85	4.9	5.5	6.7	5.6
37	5.82	4.2	4.9	6.2	5.4
38	6.36	6.3	6.0	5.6	6.4
39	5.58	5.6	5.8	5.6	5.9
40	6.88	6.1	5.8	5.6	6.0
41	6.90	8.3	6.8	6.0	6.5
42	6.72	7.0	7.0	6.3	6.3
43	5.59	5.0	5.6	5.0	5.8
44	3.54	6.3	5.9	4.7	6.1
45	2.98	2.8	2.9	3.5	3.5
46	2.50	3.2	2.9	3.0	3.4
47	3.17	1.9	2.7	2.0	2.6
48	2.87	1.8	2.3	1.5	2.2
49	3.87	2.2	1.7	1.6	2.1
50	2.51	2.5	1.8	1.7	2.3
51	2.30	2.2	1.8	1.9	2.0
52	2.34	2.3	3.1	2.0	2.2
NY	31	30	30	30	250

Table 13 Standard deviations of inflows to Mjölkvattnet from the historic and some synthetic series. Unit: VE

Week	Historic series	Synthetic serie 1	Synthetic serie 2	Synthetic serie 3	Synthetic serie 4
1	2.8	1.4	1.3	4.9	2.2
2	3.1	1.0	2.5	1.0	2.3
3	1.0	0.9	0.8	1.2	3.2
4	0.9	1.0	0.9	1.0	2.1
5	1.2	1.0	2.2	1.9	2.2
6	1.0	1.3	1.8	4.3	2.4
7	1.0	4.3	1.0	1.0	2.2
8	1.2	3.9	2.7	4.8	2.5
9	0.9	1.9	1.0	1.4	2.7
10	3.0	6.9	3.4	1.7	3.9
11	1.3	1.1	1.2	2.0	2.4
12	2.5	3.9	4.7	1.1	2.8
13	1.7	3.6	2.2	1.4	2.4
14	1.7	4.6	4.7	1.1	2.6
15	1.3	3.1	3.3	1.4	2.4
16	2.2	1.5	3.3	2.2	3.2
17	3.2	3.8	5.8	3.3	3.7
18	6.1	4.3	14.1	5.0	7.3
19	7.8	11.2	15.6	11.9	13.4
20	10.8	13.9	12.3	15.1	13.0
21	9.1	8.2	9.2	9.4	10.6
22	12.3	10.2	14.9	14.3	13.2
23	10.3	11.0	9.0	10.5	11.1
24	7.1	8.7	12.0	8.7	11.2
25	8.2	12.1	6.7	8.6	9.5
26	8.8	9.8	7.7	8.6	8.4
27	7.0	5.6	5.4	5.4	6.2
28	4.8	5.3	5.6	5.1	5.7
29	4.0	8.2	5.1	5.8	6.1
30	3.6	5.5	5.9	6.6	5.2
31	4.2	3.2	4.4	5.5	5.4
32	2.9	5.9	3.9	5.9	4.7
33	4.6	4.3	4.5	4.2	4.0
34	2.5	4.1	3.6	3.6	3.8
35	3.9	4.5	2.6	2.6	4.9
36	4.4	4.4	5.0	6.9	5.0
37	3.6	3.5	4.1	5.2	4.3
38	4.9	5.8	5.2	3.8	5.7
39	3.6	4.5	4.0	3.9	4.4
40	5.4	4.3	4.4	3.7	5.4
41	3.9	7.2	6.0	4.8	5.5
42	5.1	4.8	5.5	4.9	4.7
43	4.9	4.1	5.3	4.6	4.5
44	2.8	5.2	5.1	4.1	4.8
45	1.7	1.6	2.9	4.0	3.5
46	1.4	3.6	4.4	3.3	4.3
47	3.7	1.2	4.3	1.5	3.1
48	1.8	1.1	2.6	1.1	2.0
49	5.2	2.2	1.2	1.5	3.0
50	1.7	2.5	1.1	1.3	3.0
51	1.0	2.5	1.3	1.7	2.0
52	1.0	1.5	4.8	1.0	2.1
NY	31	30	30	30	250

Table 14 Means of weekly inflows to Korsvattnet from the historic and some generated series. Unit: VE

Week	Historic series	Synthetic serie 1	Synthetic serie 2	Synthetic serie 3	Synthetic serie 4
1	0.9	0.8	0.9	0.9	1.0
2	1.3	0.9	0.7	1.0	1.0
3	1.0	0.8	0.9	0.7	1.0
4	0.9	0.6	1.0	0.6	1.0
5	0.8	0.9	0.7	0.7	0.9
6	0.7	0.8	1.0	0.8	1.1
7	0.8	0.9	0.8	0.9	0.9
8	0.7	1.1	0.9	1.0	1.0
9	1.0	1.0	1.1	1.2	1.0
10	0.9	0.9	1.3	1.2	1.1
11	1.1	0.8	1.1	1.1	1.0
12	0.9	1.1	1.2	0.8	1.0
13	1.1	1.2	1.0	0.7	1.1
14	0.7	1.4	1.2	0.7	1.1
15	1.0	1.0	1.0	0.9	0.9
16	1.2	1.1	1.0	1.1	1.0
17	1.2	1.6	1.1	0.9	1.1
18	3.0	3.2	2.6	2.6	2.8
19	6.4	10.3	8.1	6.5	7.7
20	9.7	8.2	11.0	10.0	10.6
21	11.2	11.0	13.2	11.9	12.8
22	14.8	13.9	15.8	16.4	15.7
23	14.6	12.8	15.3	14.2	13.8
24	12.7	12.0	11.9	11.5	12.0
25	10.9	12.7	9.1	10.5	10.3
26	8.8	10.7	7.7	7.6	8.4
27	6.1	5.5	4.8	5.1	5.3
28	4.6	3.8	3.6	4.4	3.7
29	3.7	3.5	2.9	3.7	3.3
30	3.3	3.5	2.9	4.3	3.4
31	3.1	3.4	2.6	3.8	3.1
32	3.0	3.7	2.5	3.2	3.0
33	2.1	2.9	2.6	3.9	2.9
34	1.8	3.0	2.6	3.5	2.9
35	2.6	2.8	2.8	2.6	2.8
36	3.2	3.1	3.1	3.3	3.2
37	3.5	2.2	2.6	3.6	3.0
38	3.2	3.0	3.2	3.6	3.2
39	3.1	3.0	3.5	3.2	3.2
40	4.1	3.3	2.7	3.1	3.1
41	3.8	3.5	3.0	2.8	3.1
42	3.5	3.6	3.6	3.6	3.3
43	3.1	2.7	3.2	2.9	3.1
44	1.9	3.3	2.9	3.1	3.2
45	1.8	2.2	1.8	1.9	2.0
46	1.2	1.5	1.3	1.8	1.6
47	1.7	1.1	1.6	0.9	1.3
48	1.6	1.0	1.1	0.7	1.1
49	1.5	1.0	1.0	0.7	1.0
50	1.2	1.0	1.1	0.8	1.1
51	0.9	1.0	0.9	0.8	1.0
52	1.0	1.3	1.2	1.1	1.2
NY	29	30	30	30	250

Table 15 Standard deviations of weekly inflows to Korsvattnet from the historic and some generated series. Unit: VE

Week	Historic series	Synthetic serie 1	Synthetic serie 2	Synthetic serie 3	Synthetic serie 4
1	1.0	0.7	0.8	0.9	0.8
2	1.9	0.9	0.5	0.8	0.9
3	0.8	0.7	0.7	0.6	1.3
4	0.7	0.6	0.8	0.5	0.9
5	0.9	1.4	0.6	0.6	0.9
6	0.6	0.9	1.8	0.7	1.5
7	0.6	1.1	0.7	0.8	0.9
8	0.6	1.4	1.1	0.8	1.1
9	0.9	0.8	0.9	1.9	1.0
10	1.1	1.2	1.6	1.2	1.5
11	1.2	0.7	0.7	0.9	1.1
12	0.9	1.2	1.6	0.7	1.0
13	1.1	1.2	1.0	1.0	1.1
14	0.7	1.2	1.4	0.6	1.2
15	0.7	0.8	1.0	0.7	0.9
16	1.0	0.9	0.9	0.8	0.9
17	1.0	1.2	0.8	0.7	0.9
18	3.1	3.7	2.2	2.8	3.0
19	4.4	11.0	9.0	4.7	8.6
20	6.5	8.2	6.6	6.6	6.2
21	5.5	5.9	5.4	5.1	6.3
22	7.7	7.3	6.6	7.1	7.0
23	6.2	5.0	5.5	5.6	5.3
24	5.0	6.5	5.4	5.0	5.5
25	5.4	7.6	4.2	5.5	5.5
26	5.5	6.8	4.3	5.3	5.4
27	3.9	3.0	3.6	3.7	3.6
28	2.7	2.7	3.4	3.4	2.9
29	3.1	3.1	2.6	3.4	2.8
30	2.3	2.5	2.8	3.6	3.0
31	2.8	2.8	2.7	3.8	2.9
32	2.1	2.9	1.7	3.3	2.9
33	1.4	1.7	2.5	3.8	2.6
34	1.5	3.0	2.3	2.4	2.8
35	1.8	2.5	1.9	1.8	2.2
36	2.6	2.8	2.1	2.5	2.4
37	2.3	2.0	2.1	2.5	2.2
38	2.1	2.1	2.6	2.4	2.2
39	1.8	2.0	2.4	2.4	2.3
40	2.7	2.6	1.8	2.4	2.5
41	2.1	2.8	2.2	2.6	2.3
42	2.4	2.1	2.7	2.5	2.2
43	2.5	2.0	2.7	2.7	2.3
44	1.2	2.5	1.8	2.5	2.1
45	1.1	2.4	2.3	1.8	2.3
46	1.0	1.6	1.1	2.1	1.5
47	1.6	1.4	1.9	0.8	1.6
48	1.6	0.6	1.2	0.8	0.9
49	1.6	0.9	0.9	0.6	0.9
50	0.9	0.9	1.5	0.9	1.1
51	0.7	1.0	1.3	0.7	1.0
52	0.8	1.4	1.7	0.8	1.2
NY	29	30	30	30	250

Table 16 Some parameters of inflow to Mjölkvattnet from the historic and some generated series. Unit: VE

	Historic series	Synthetic serie 1	Synthetic serie 2	Synthetic serie 3	Synthetic serie 4	
Mean	314.67	380.80	315.73	318.26	321.26	
Stand. dev.	59.14	66.53	60.21	54.27	57.55	yearly
γ_1	0.23	0.63	0.04	0.38	0.07	sums
γ_2	2.86	3.30	3.36	4.46	2.92	
Mean	168.45	169.71	169.49	165.70	167.93	QV
Stand. dev.	33.35	31.83	29.16	23.94	28.64	
γ_1	0.25	0.47	-0.41	-0.06	-0.12	
γ_2	3.17	3.01	2.93	2.87	2.79	
Mean	31.05	36.02	34.84	32.03	33.44	exp (QH)
Stand. Dev.	14.93	19.17	14.41	12.31	14.68	(QH defined
γ_1	1.56	1.00	0.52	1.96	1.26	by ex (59))
γ_2	6.36	3.06	2.25	8.99	4.32	
NY	31	30	30	30	250	

Table 17 Some parameters of inflow to Korsvattnet from the historic and some generated series. Unit: VE

	Historic series	Synthetic serie 1	Synthetic serie 2	Synthetic serie 3	Synthetic serie 4	
Mean	178.97	181.68	176.76	179.13	180.21	yearly
Stand. dev.	32.87	28.22	31.81	26.53	29.76	sums
γ_1	0.37	1.03	0.03	-0.32	0.03	
γ_2	3.78	3.99	4.02	3.98	3.04	
Mean	95.31	98.40	97.73	94.16	97.16	QV
Stand. dev.	18.46	15.28	15.13	13.15	15.73	
γ_1	0.18	0.16	-0.38	0.07	-0.08	
γ_2	2.73	3.65	2.76	3.00	2.77	
Mean	20.54	19.96	20.96	17.72	20.77	
Stand. dev.	10.47	8.39	8.82	6.68	8.35	QH
γ_1	0.79	1.22	0.53	-0.08	0.68	
γ_2	3.75	5.67	2.55	2.83	3.16	
NY	29	30	30	30	250	

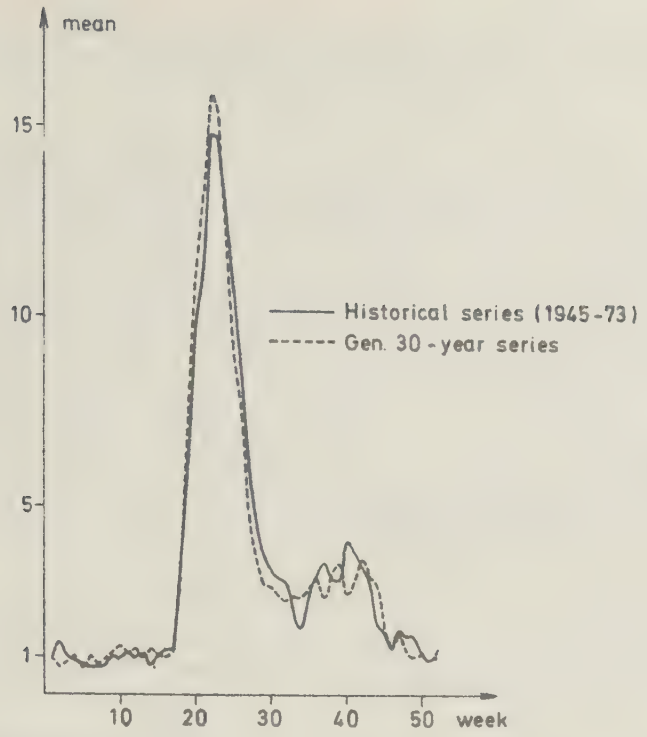

Fig 7 Means of weekly inflow to Korsvattnet from the historical series and synthetic series 2. Unit: VE

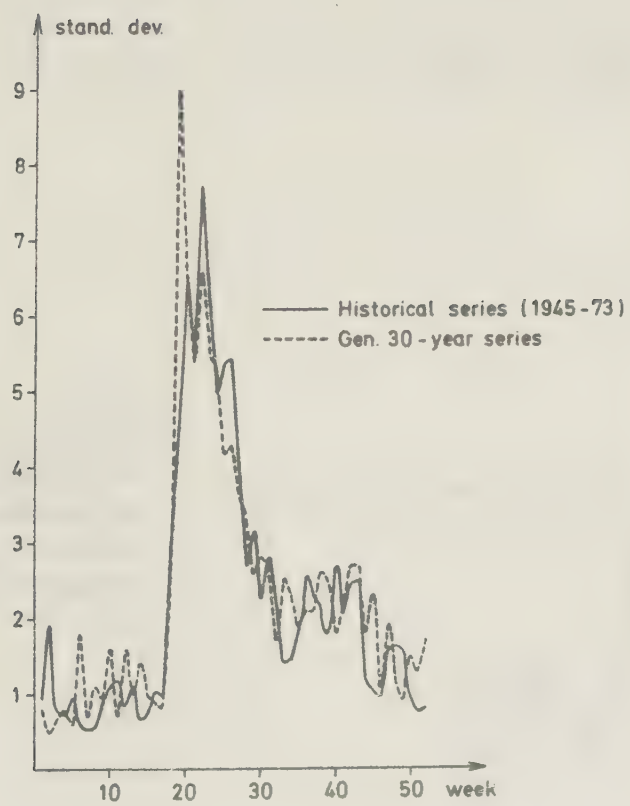

Fig 8 Standard deviations of weekly inflow to Korsvattnet from the historical series and synthetic series 2. Unit: VE

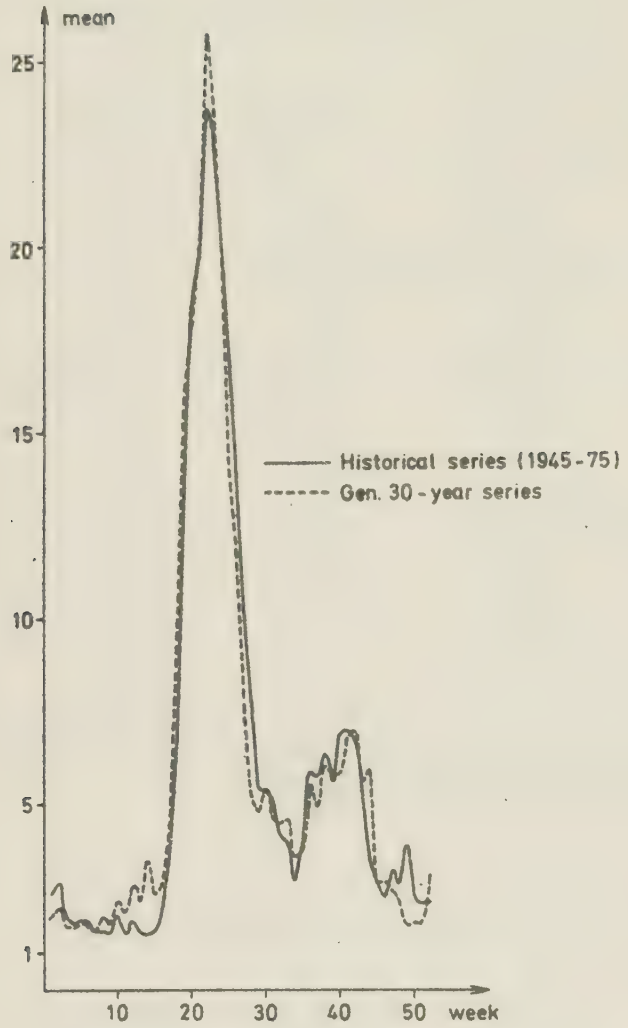

Fig 9
Means of weekly inflow to Stora Mjölkvattnet from the historical series and synthetic series 2
Unit: VE

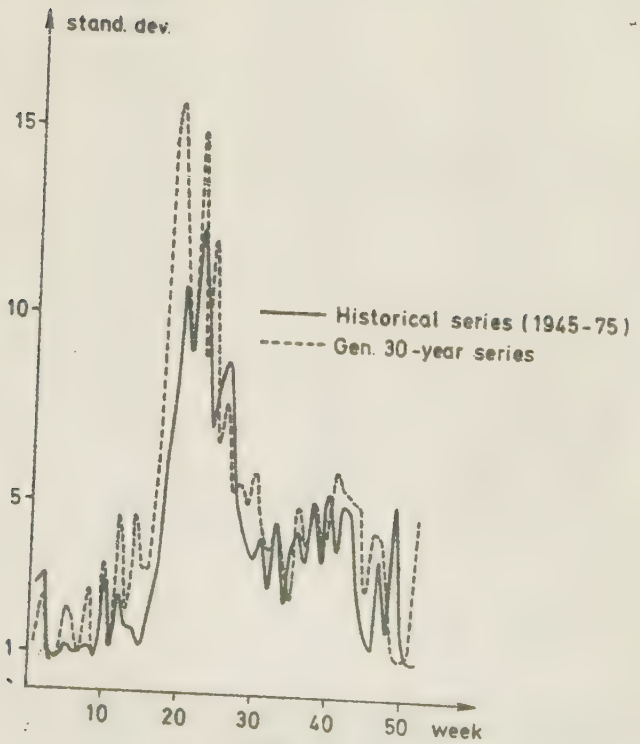

Fig 10
Standard deviations of weekly inflow to Stora Mjölkvattnet from the historical series and synthetic series 2
Unit: VE

5.8 SOME FINAL REMARKS ON THE MODELS OF KORSVATTNET AND STORA MJÖLKVATTNET

We have tried to model a process with more or less continuous seasonality by using (outside the spring-period) only 3 "seasons" in our models, and with sharp limits between them. However, this does not mean that generated Q is stationary inside these model seasons. It is trivial that the spring-season is non-stationary, but not even the other periods are stationary. Often the inflow during last week of period 2 (week 26) corresponds to a high class of period 3, hence we also expect the inflow of week 27 to be rather high, although usually not as high as in week 26. That is, the model makes it likely that on the whole the inflow decreases during season 3. But the inflows differ between different years, and many different patterns may appear during this period as well as during other periods.

In order to succeed with these models, the spring period should start somewhat earlier than the ordinary time of maximum snow-melt inflow and end after most of the snow-melt inflow usually has passed.

A critical point is the assumption of stationarity of the conditional distributions within the periods. The tests on the sequence $\bar{u}_t$ (and to some degree the tests on the sequence u_t) are an attempt to investigate this. The relation on $\bar{u}_t$ are good at least for Korsvattnet, but the problem is not fully solved.

Turning to the snow-melt model, a drawback is that QV is exactly fixed at start of the spring-period. A possible modification (besides using the same approach as in chapter 4) is to allow negative R_V and use a probability distribution of QPR also for pentade 26. This would lead to a model where QV is not exactly fixed, but just predicted.

6 REFERENCES

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SOME DEFINITIONS AND ABBREVIATIONS.

Hurst coefficient	see section 3.4.
Inflow	see chapter 1.
ML-method	Maximum-likelihood method
Unit normal distribution	see eq (4).
VE	see section 5.1.
Test A, Test B	see section 4.6.

APPENDIX: SOME PLOTS

		page
Fig 11	Historic record of pentade runoff at Ytterholmen	2(49)- 7(49)
Fig 12	Synthetic series nr 1 of pentade runoff at Ytterholmen. Cases with runoff greater than 170 units are denoted with +.	8(49)-14(49)
Fig 13	Conditional distributions for Korsvattnet.	15(49)-23(49)
Fig 14	Unconditional distributions of weekly inflows for Korsvattnet obtained from the historic and a generated 250-year series.	24(49)-36(49)
Fig 15	Historic series of weekly inflow to Stora Mjölkvattnet and Korsvattnet.	37(49)-43(49)
Fig 16	Generated bivariate series nr 2 of inflows to Stora Mjölkvattnet and Korsvattnet.	44(49)-49(49)

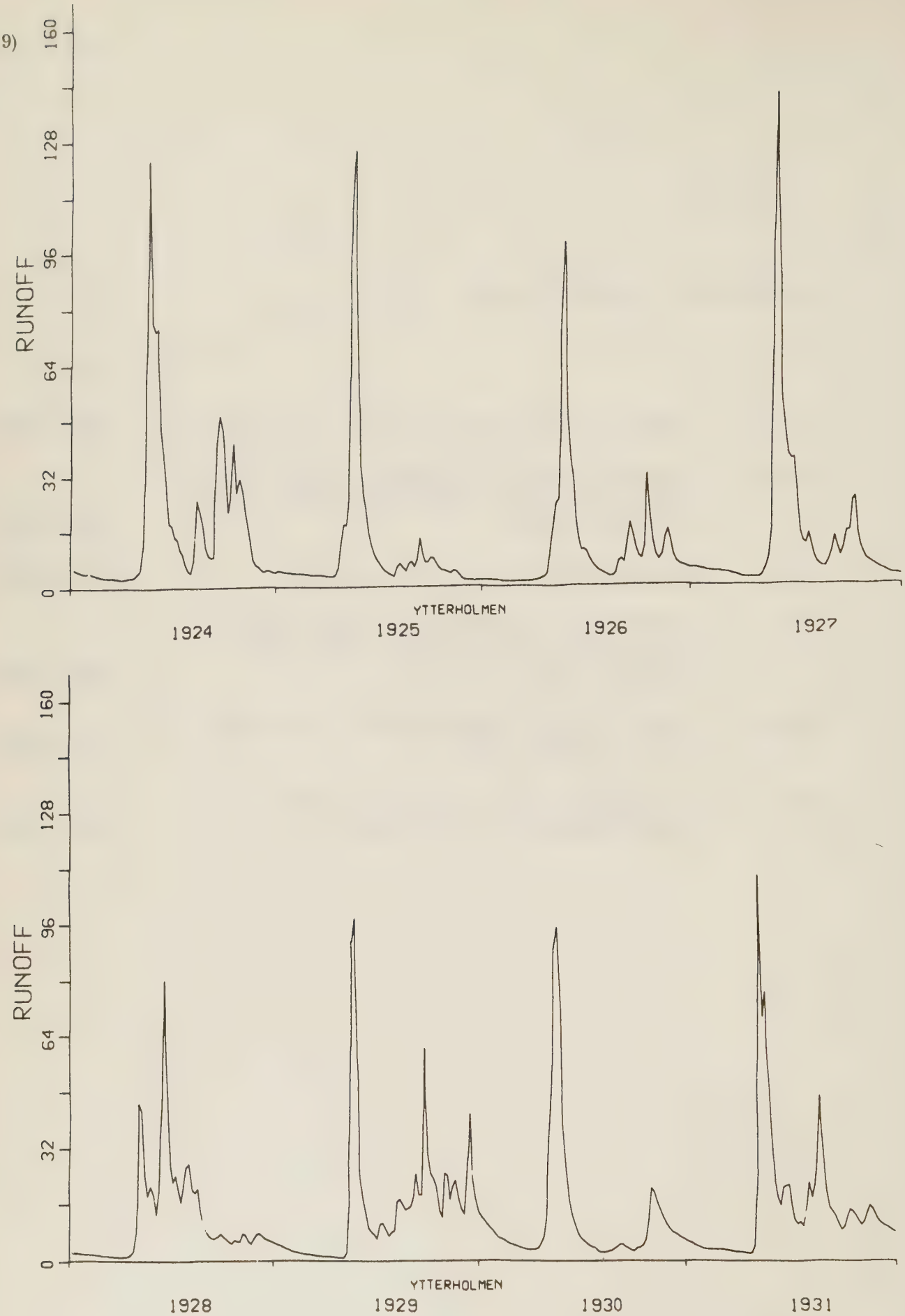
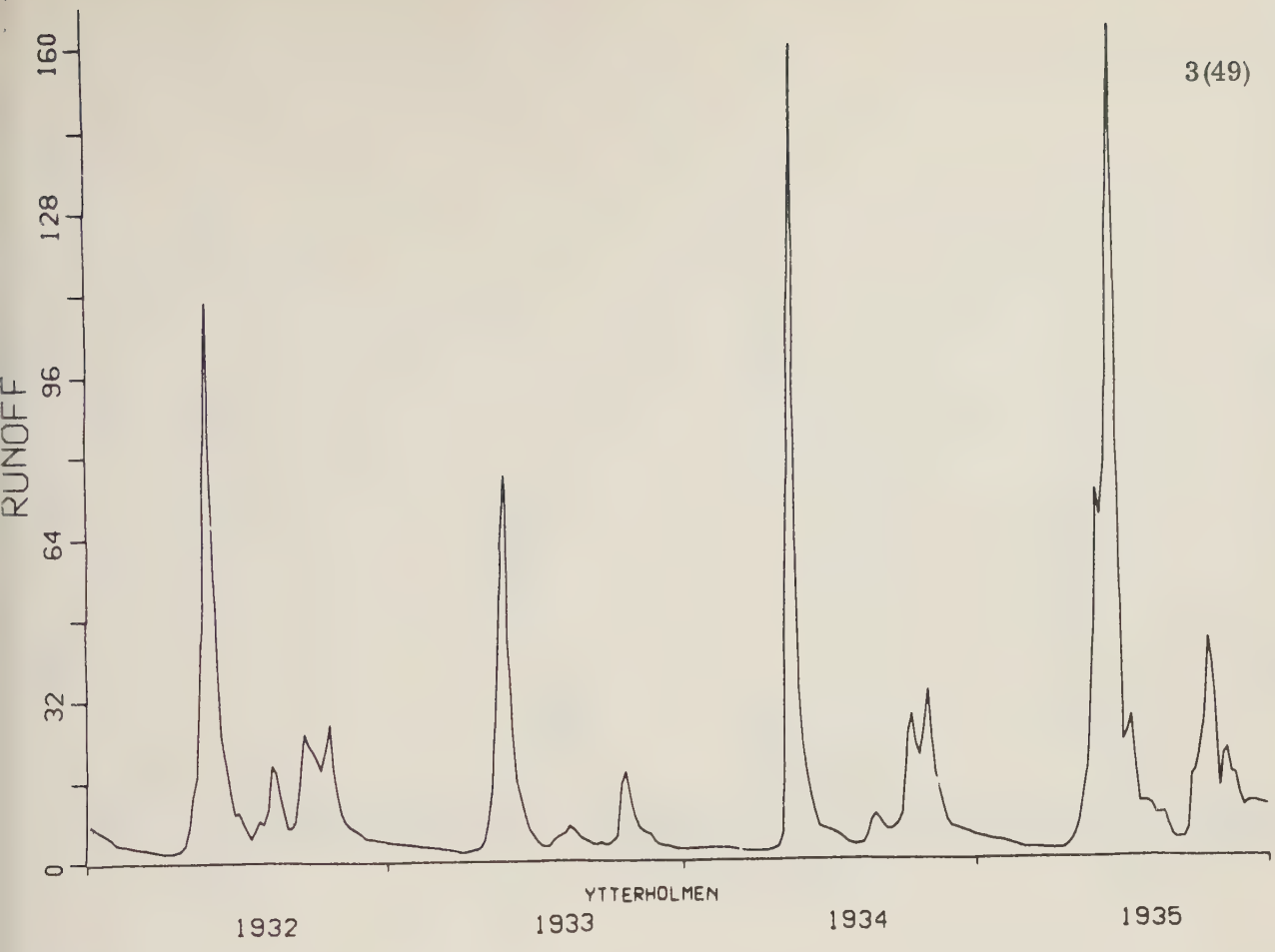
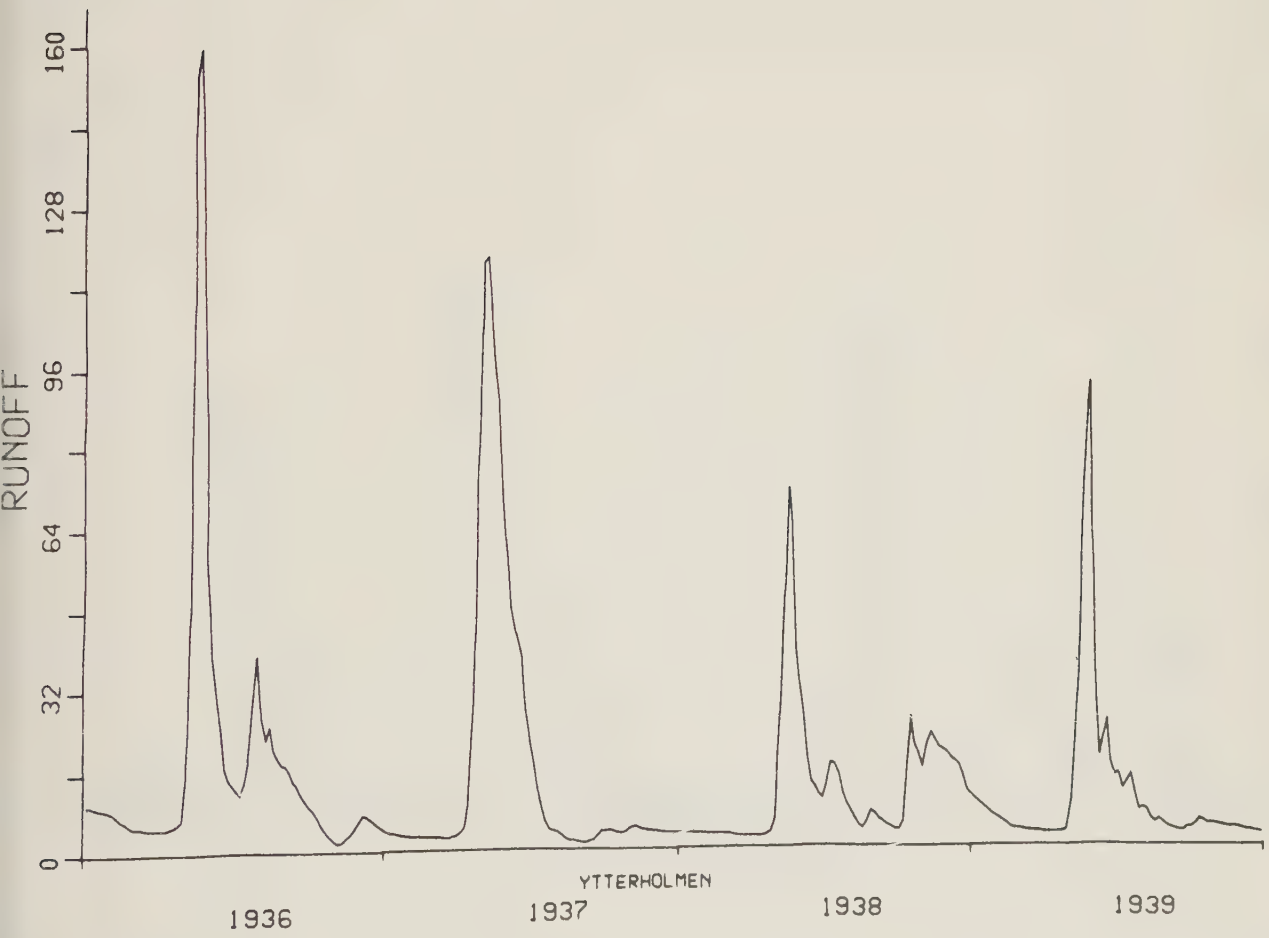


Fig 11. Historic record of pentade runoff at Ytterholmen.
Unit: 100 l/s


RUNOFF
0
32
64
96
128
160
YTTERHOLMEN
1932
1933
1934
1935


RUNOFF
0
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64
96
128
160
YTTERHOLMEN
1936
1937
1938
1939

11. cont.

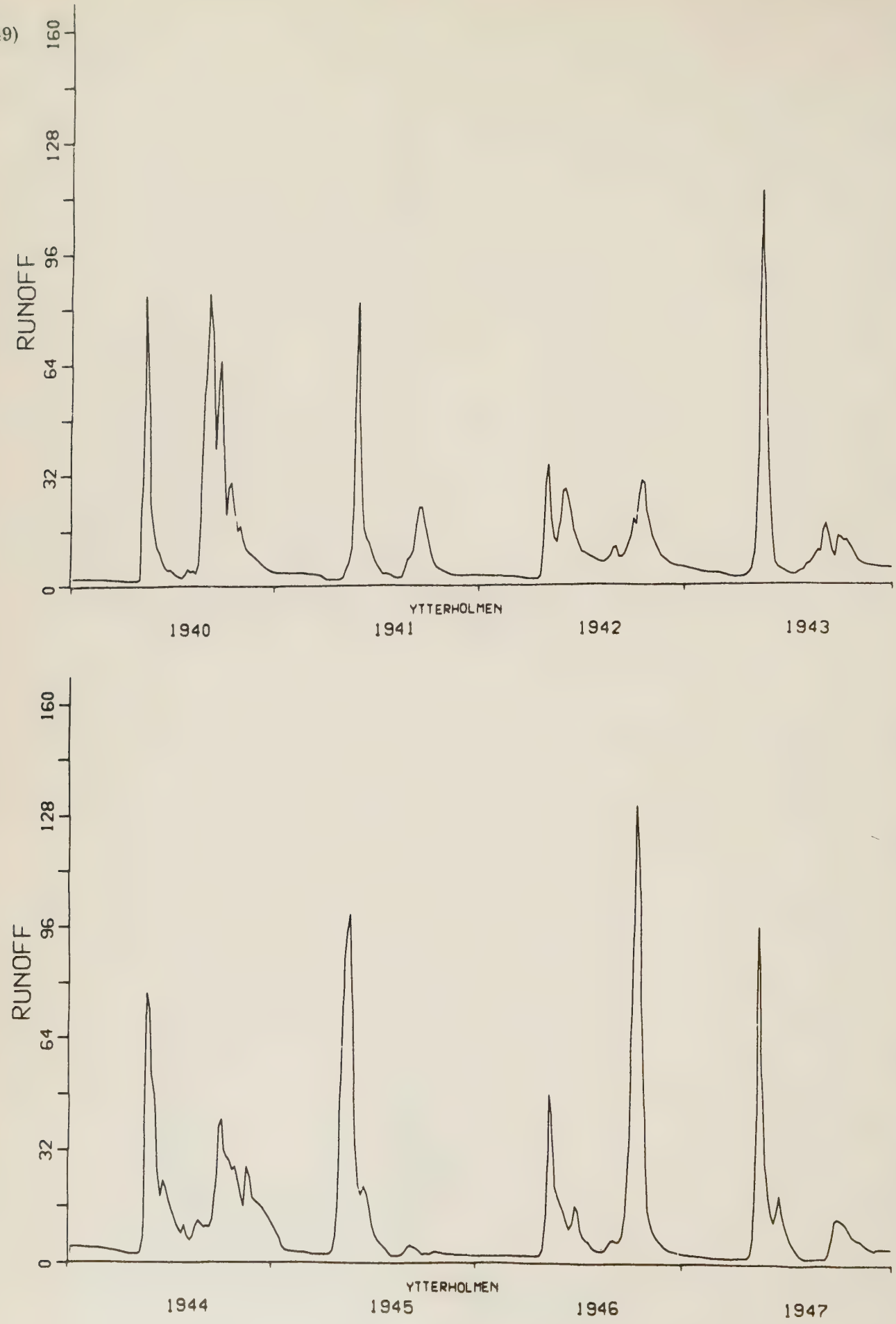
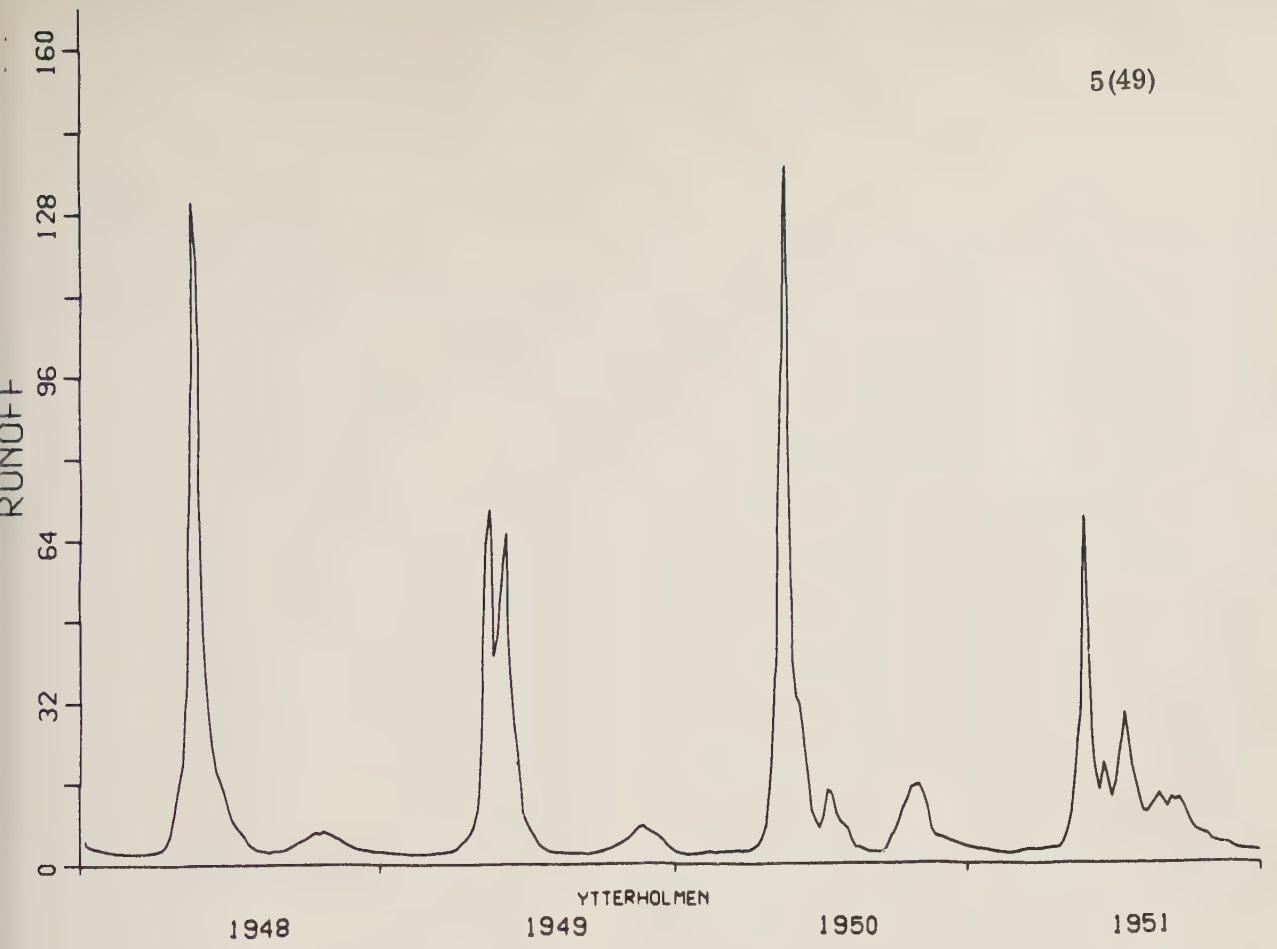
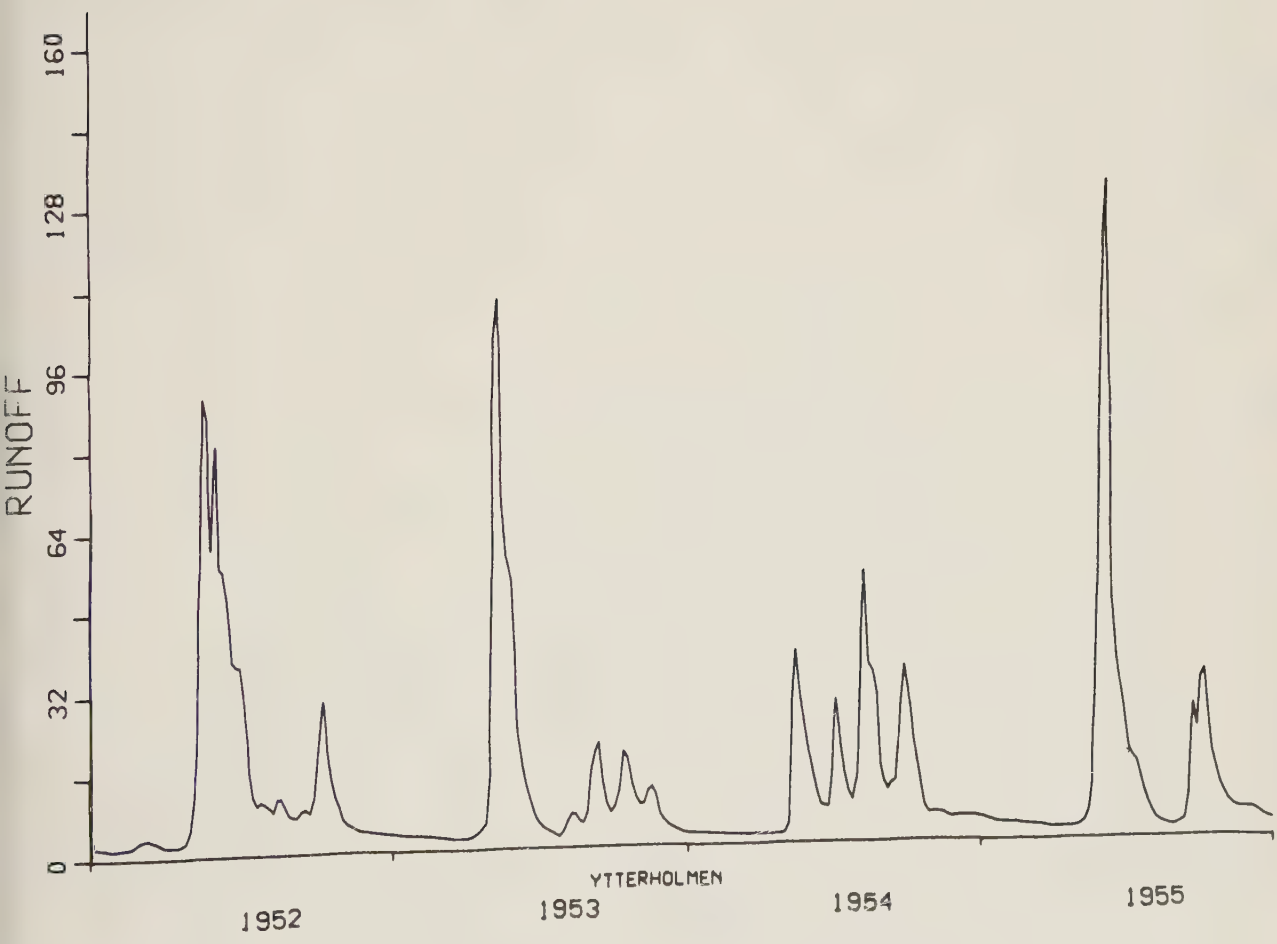
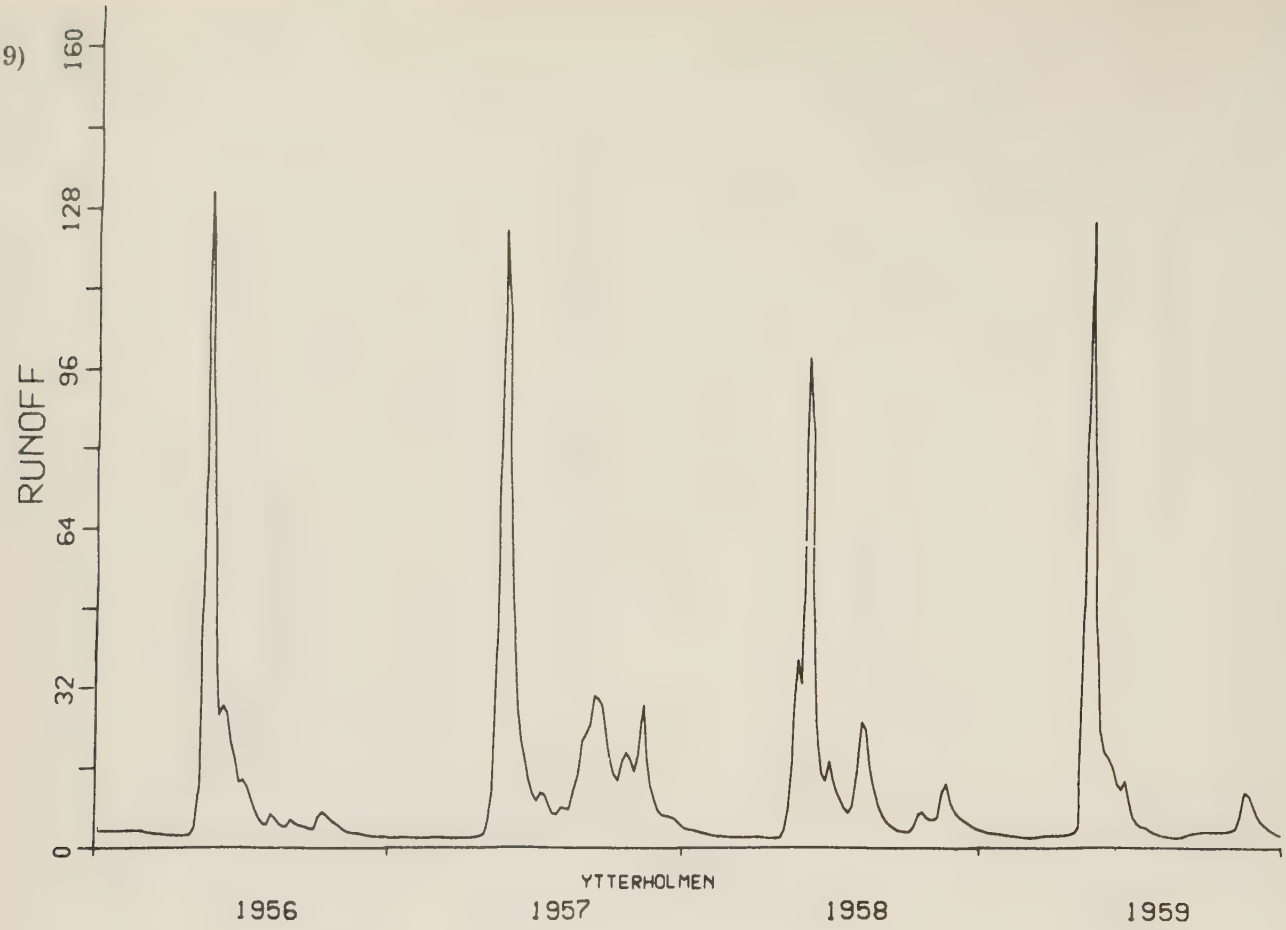

RUNOFF
0
32
64
96
128
160
YTTERHOLMEN
1940
1941
1942
1943
RUNOFF
0
32
64
96
128
160
YTTERHOLMEN
1944
1945
1946
1947

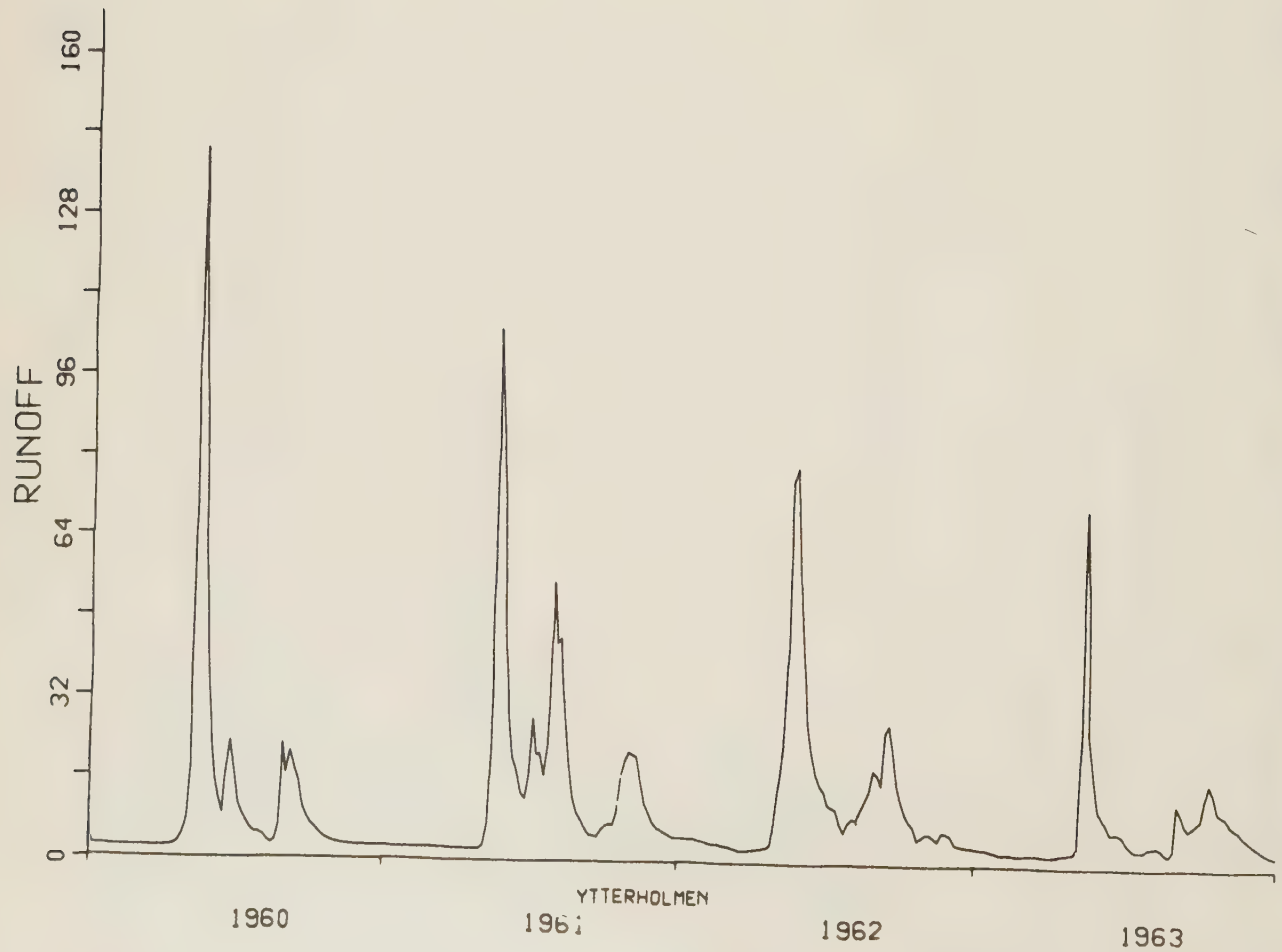
Fig 11. cont.


RUNOFF
0
32
64
96
128
160
YTTERHOLMEN
1948
1949
1950
1951


RUNOFF
0
32
64
96
128
160
YTTERHOLMEN
1952
1953
1954
1955

; 11. cont.


RUNOFF
0
32
64
96
128
160
YTTERHOLMEN
1956
1957
1958
1959


RUNOFF
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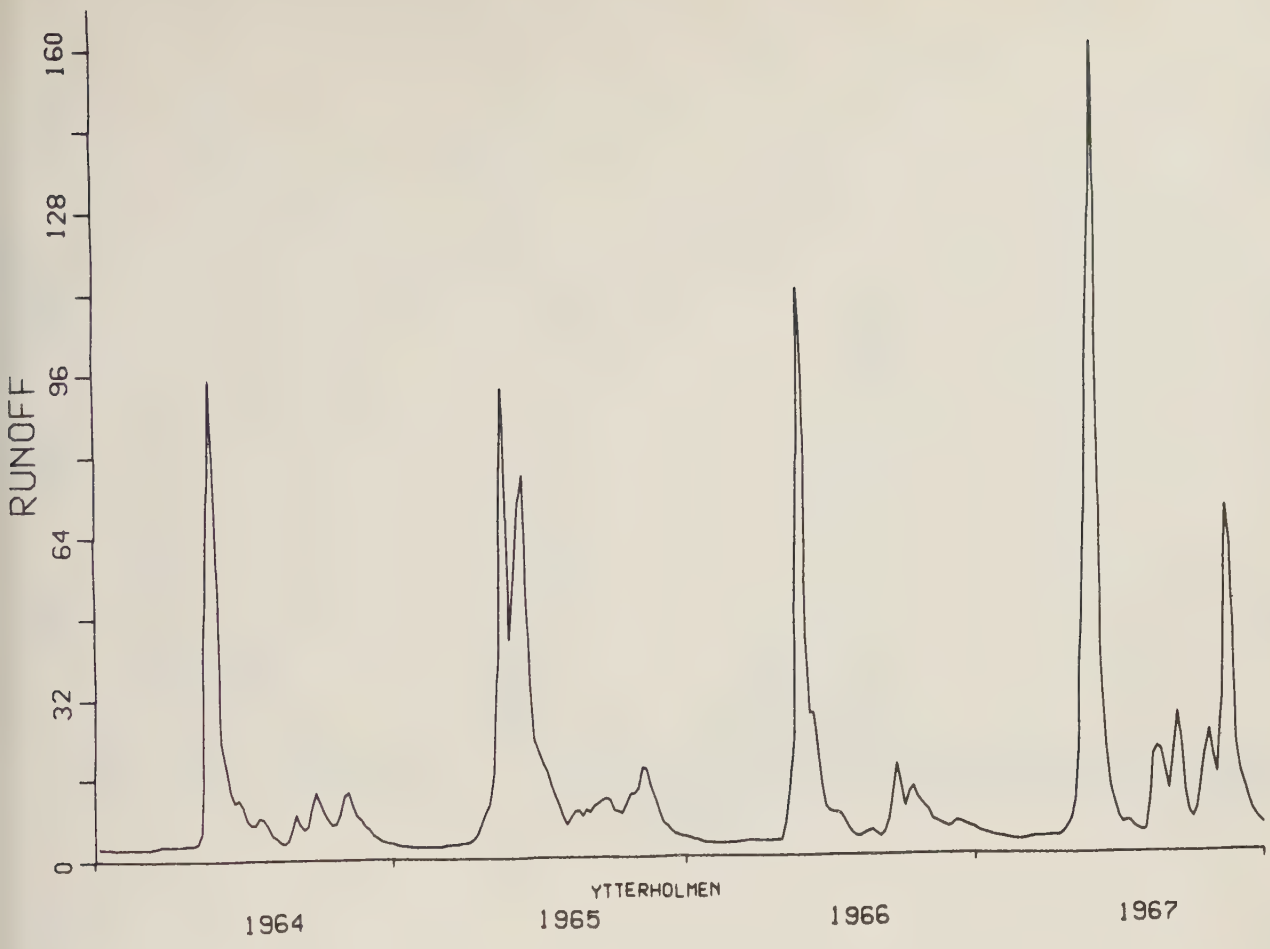
32
64
96
128
160
YTTERHOLMEN
1960
1961
1962
1963

Fig 11. cont.



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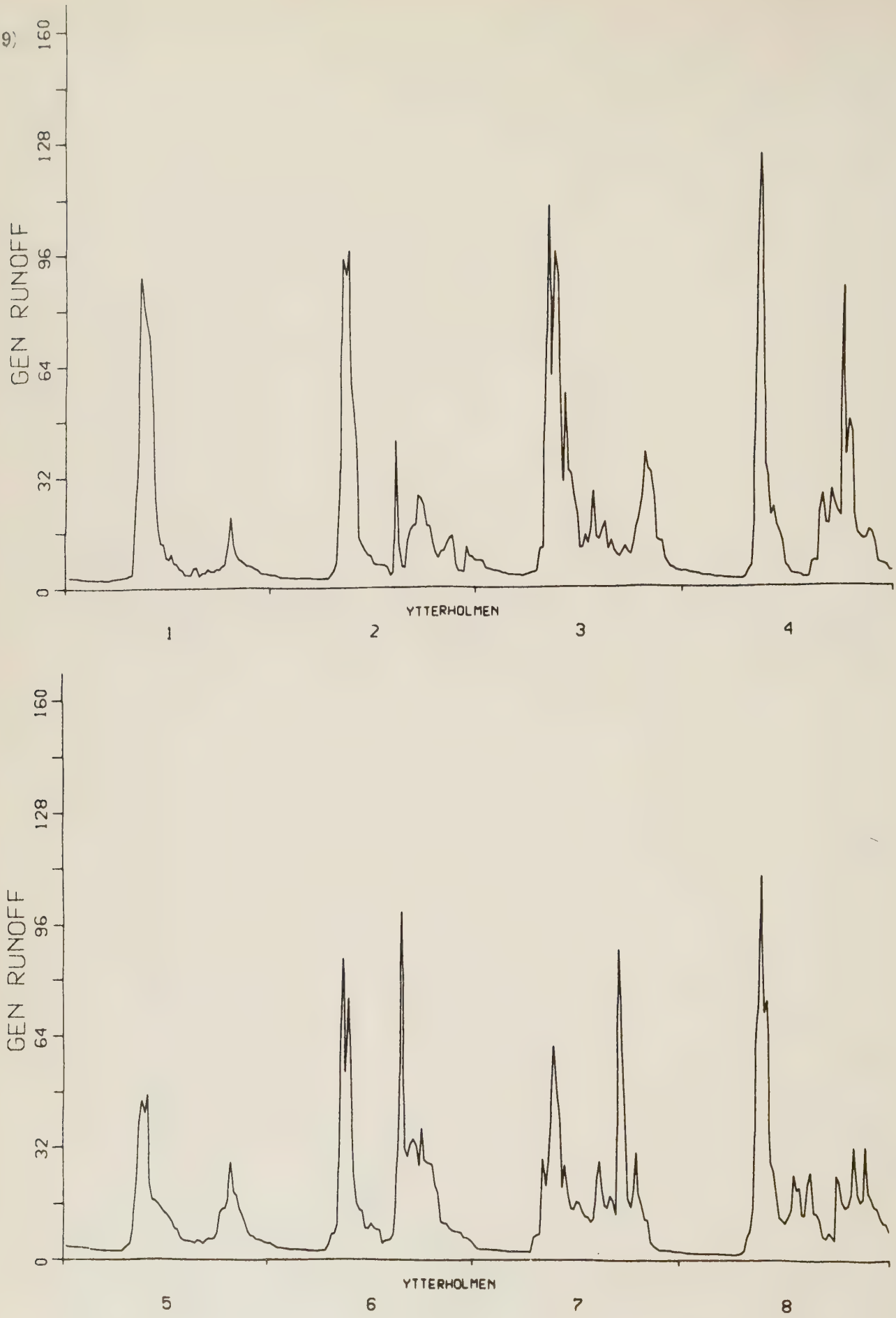


Fig 12. Synthetic series no. 1 of pentade runoff at Ytterholmen
Unit: 100 l/s

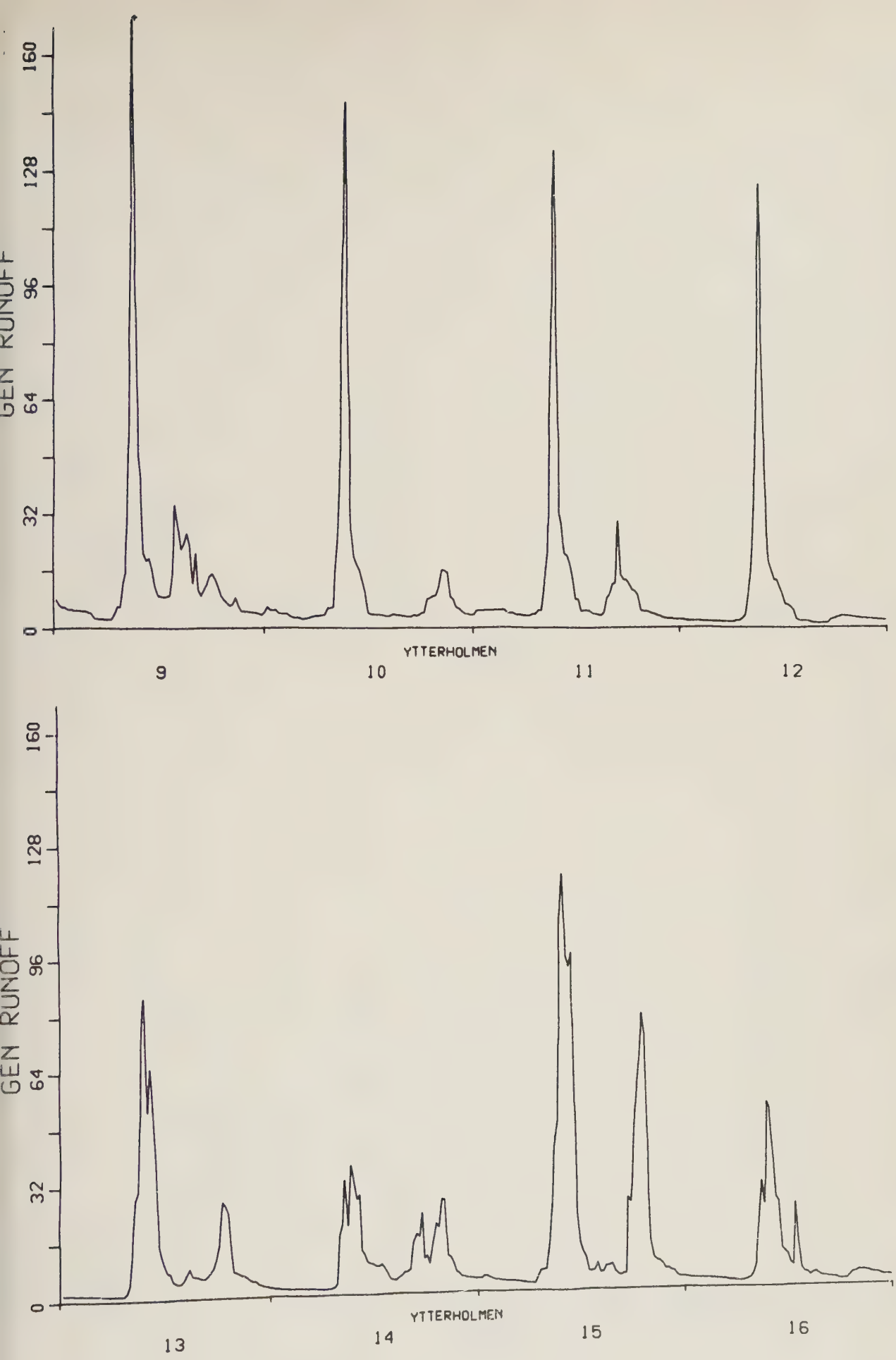

GEN RUNOFF
0
32
64
96
128
160
YTTERHOLMEN
9
10
11
12
GEN RUNOFF
0
32
64
96
128
160
YTTERHOLMEN
13
14
15
16

Fig 12. cont.

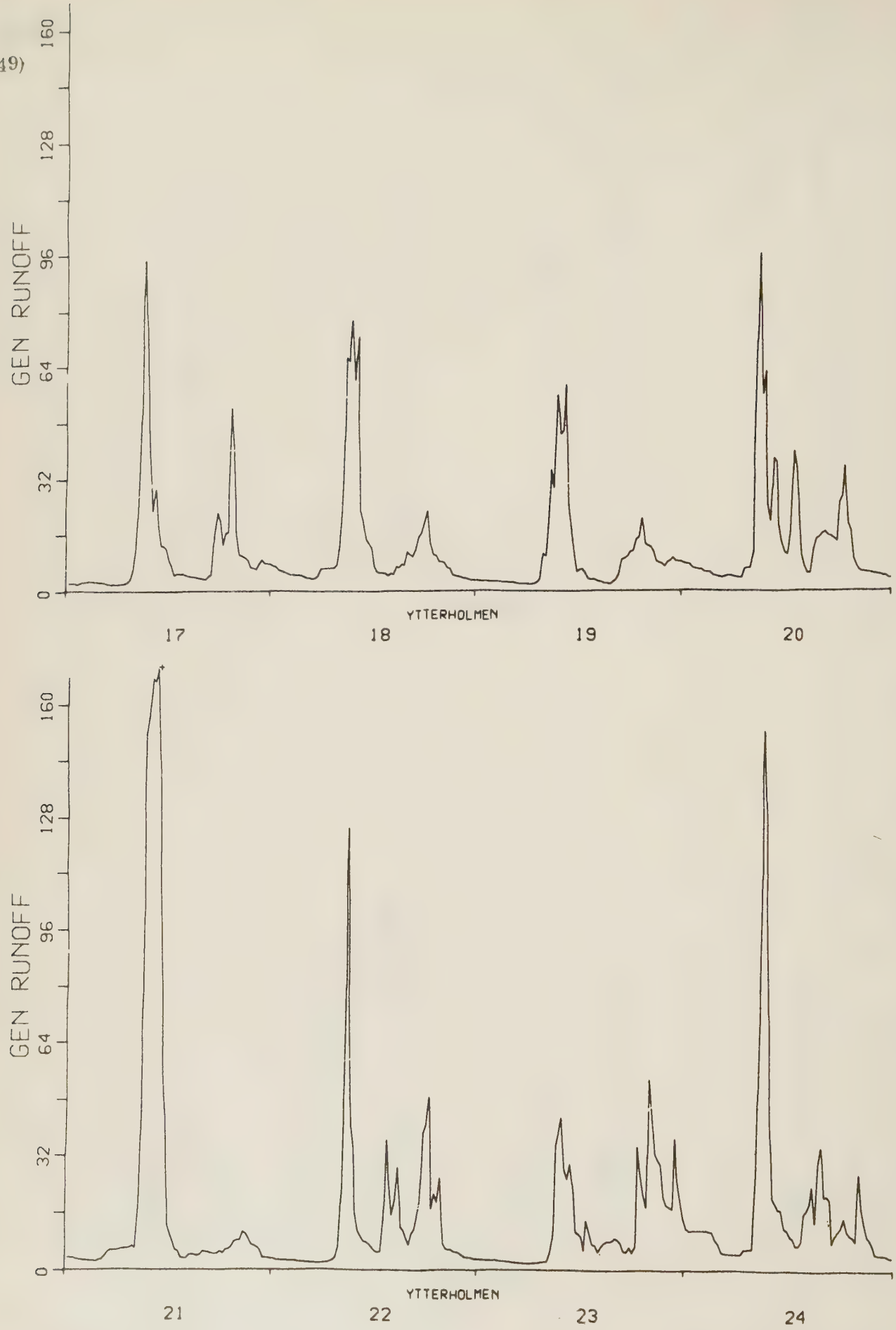
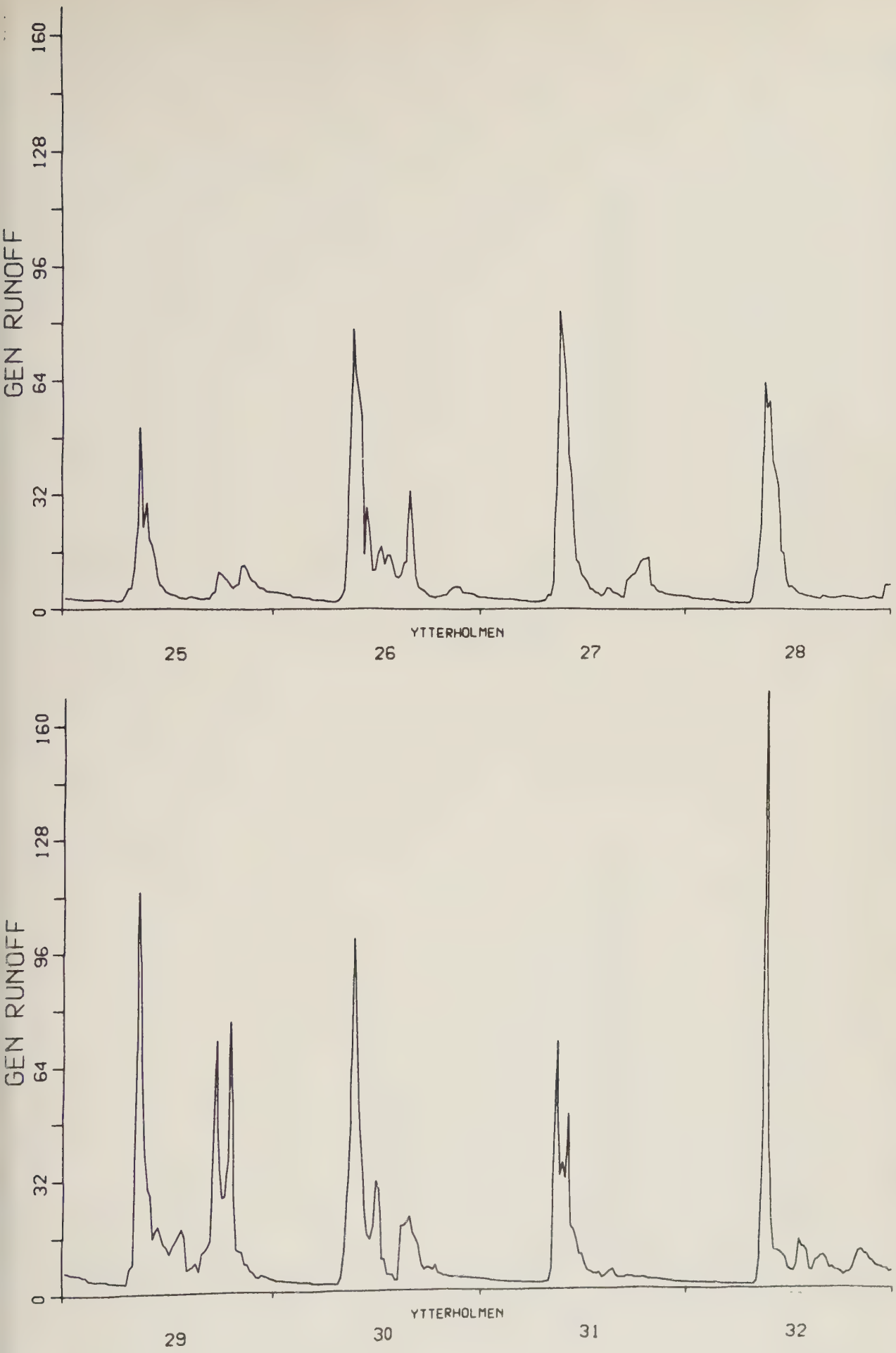

GEN RUNOFF
0
32
64
96
128
160
YTTERHOLMEN
17
18
19
20
GEN RUNOFF
0
32
64
96
128
160
YTTERHOLMEN
21
22
23
24

Fig 12. cont.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0
32
64
96
128
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YTTERHOLMEN
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GEN RUNOFF
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128
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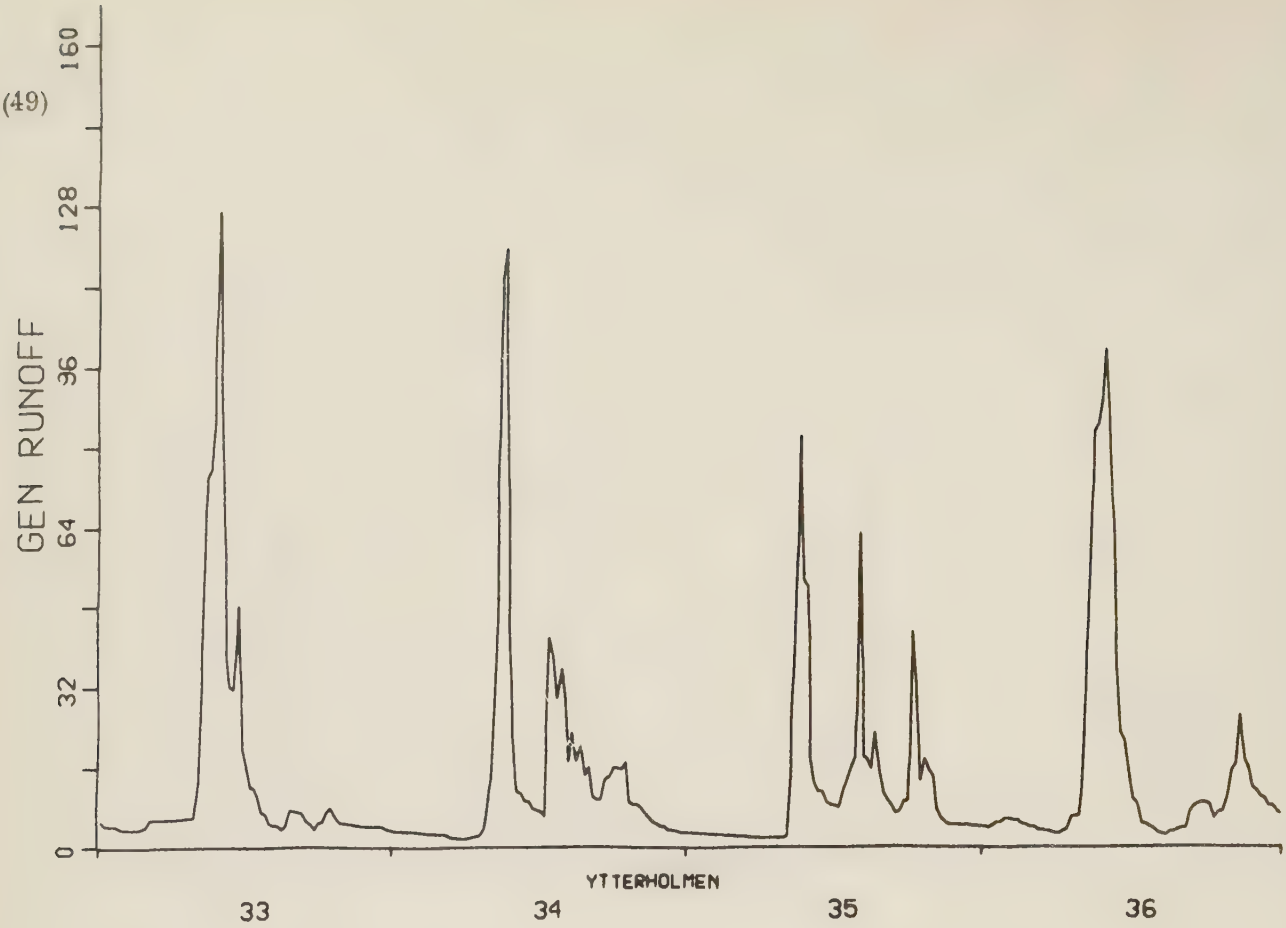
YTTERHOLMEN
29
30
31
32

Fig 12. cont.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34
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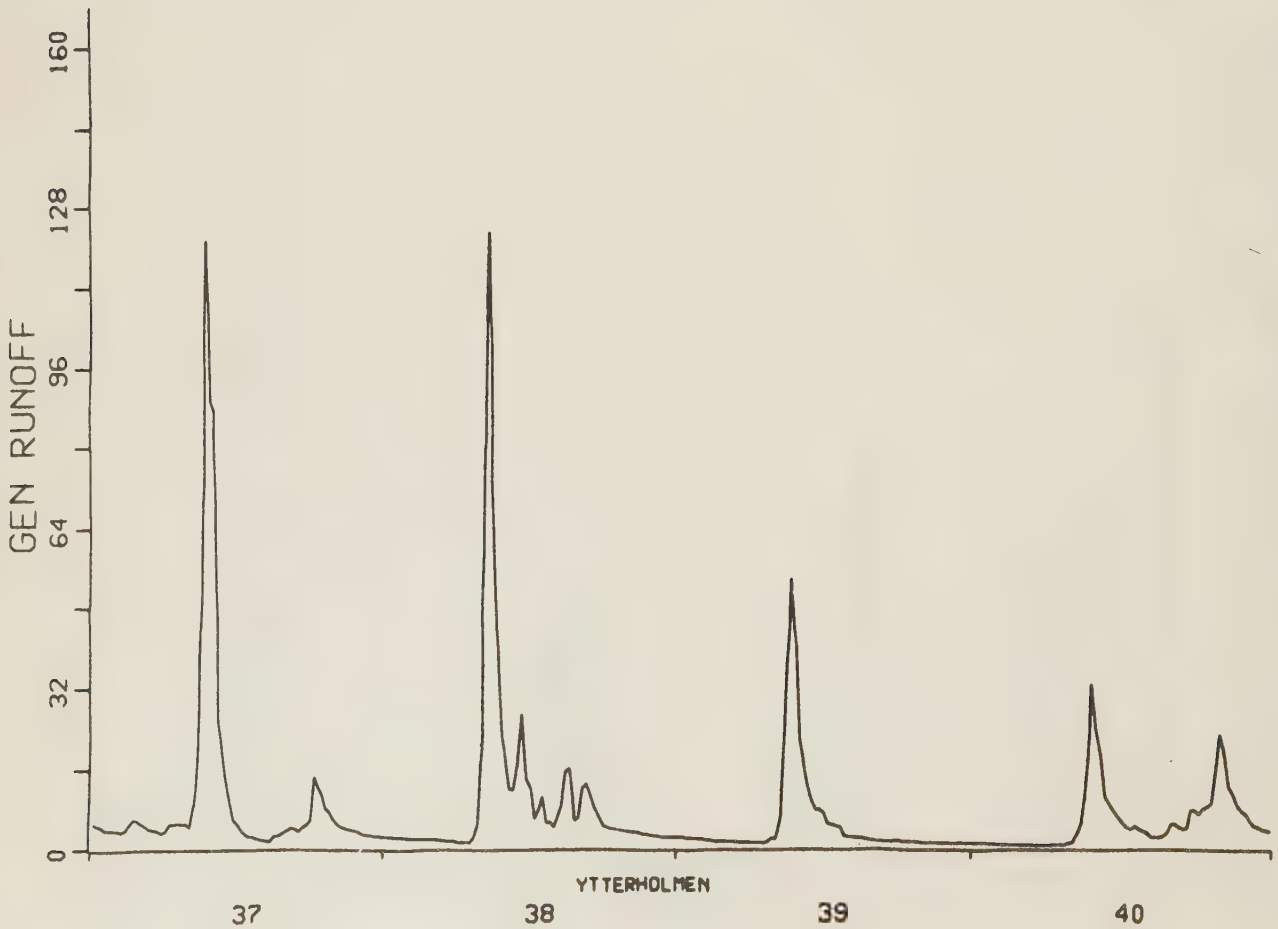
GEN RUNOFF
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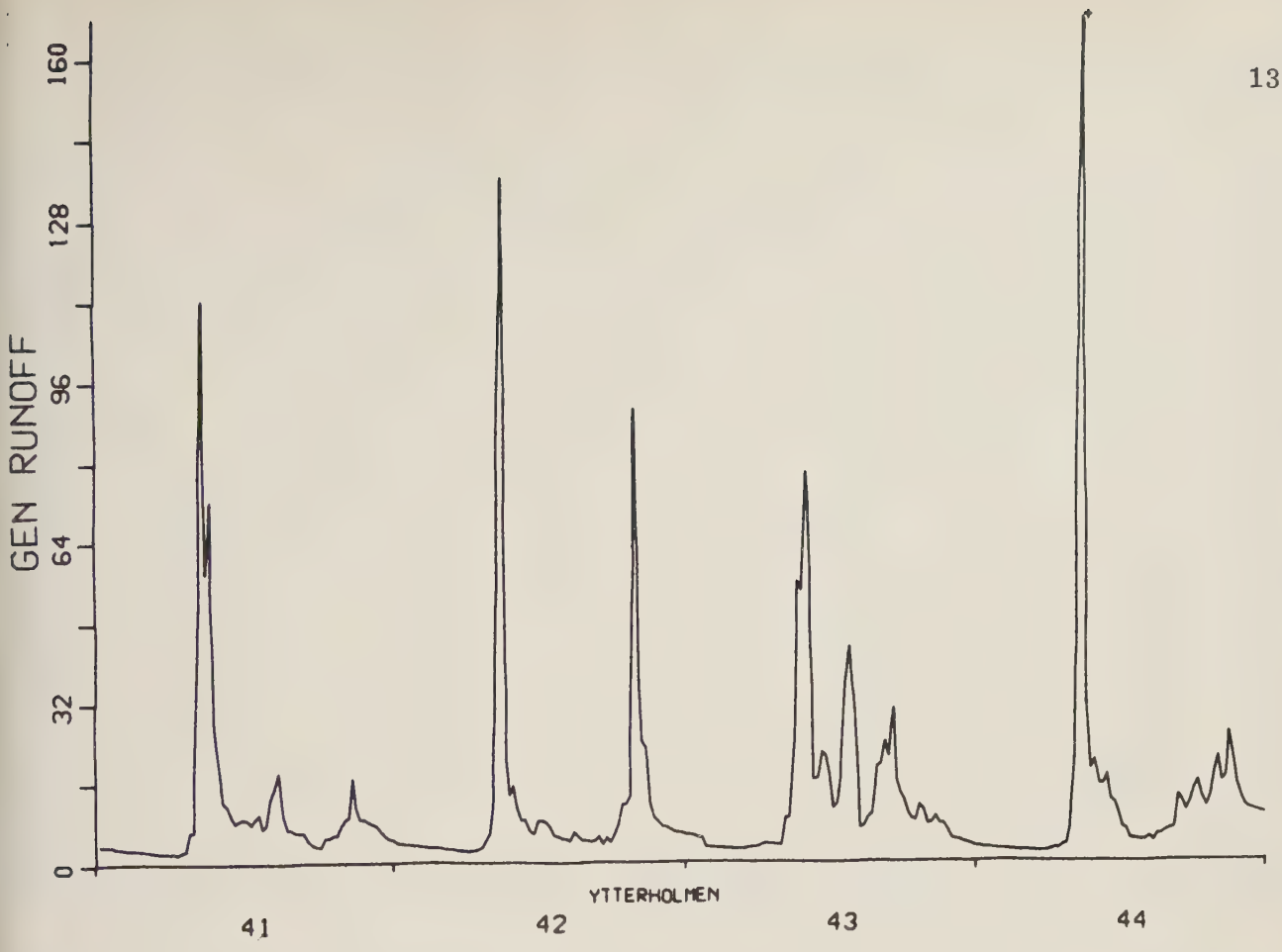
64
96
128
160
YTTERHOLMEN
37
38
39
40

Fig 12. cont.


GEN RUNOFF
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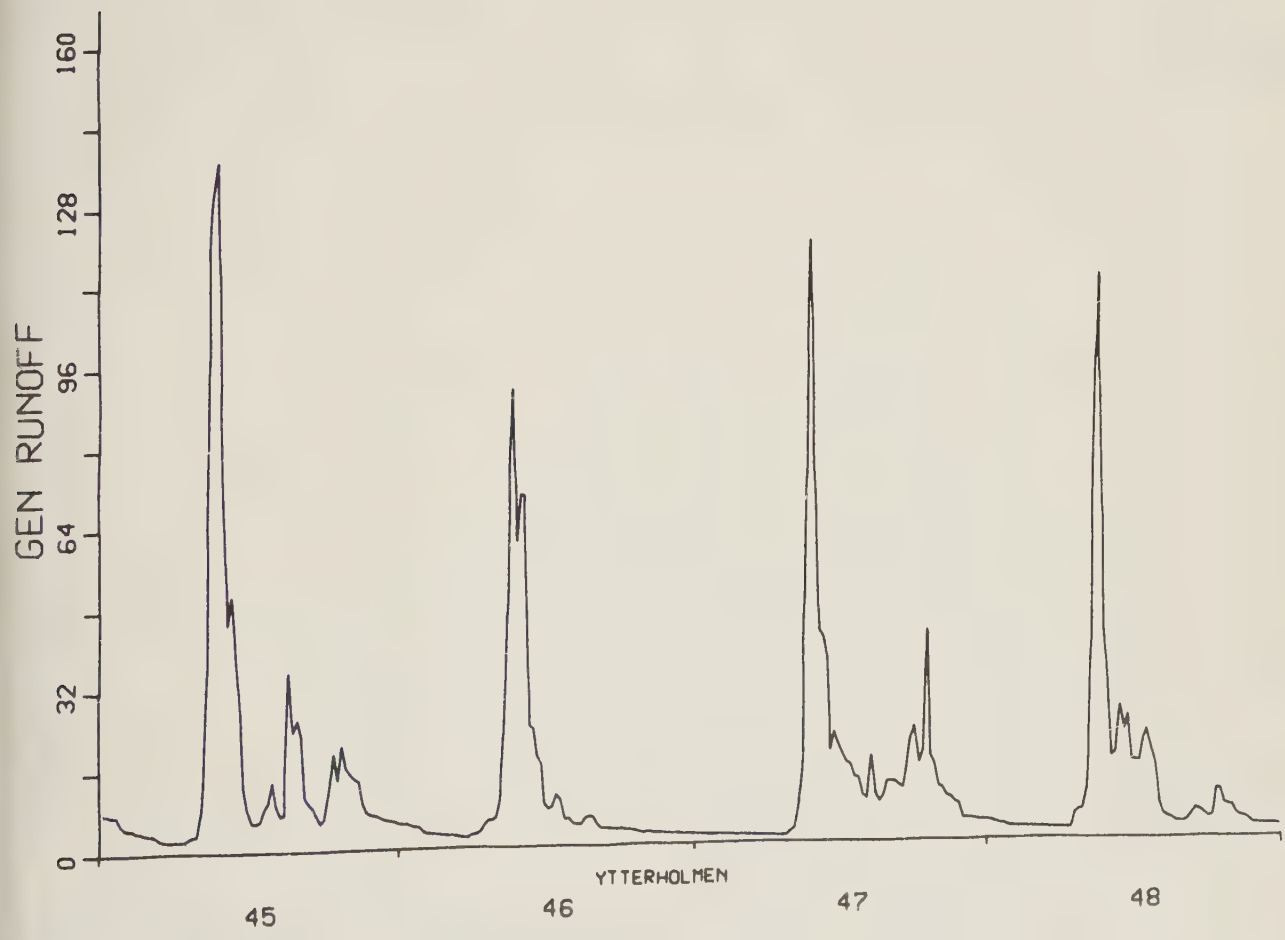
64
96
128
160
YTTERHOLMEN
41
42
43
44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64
96
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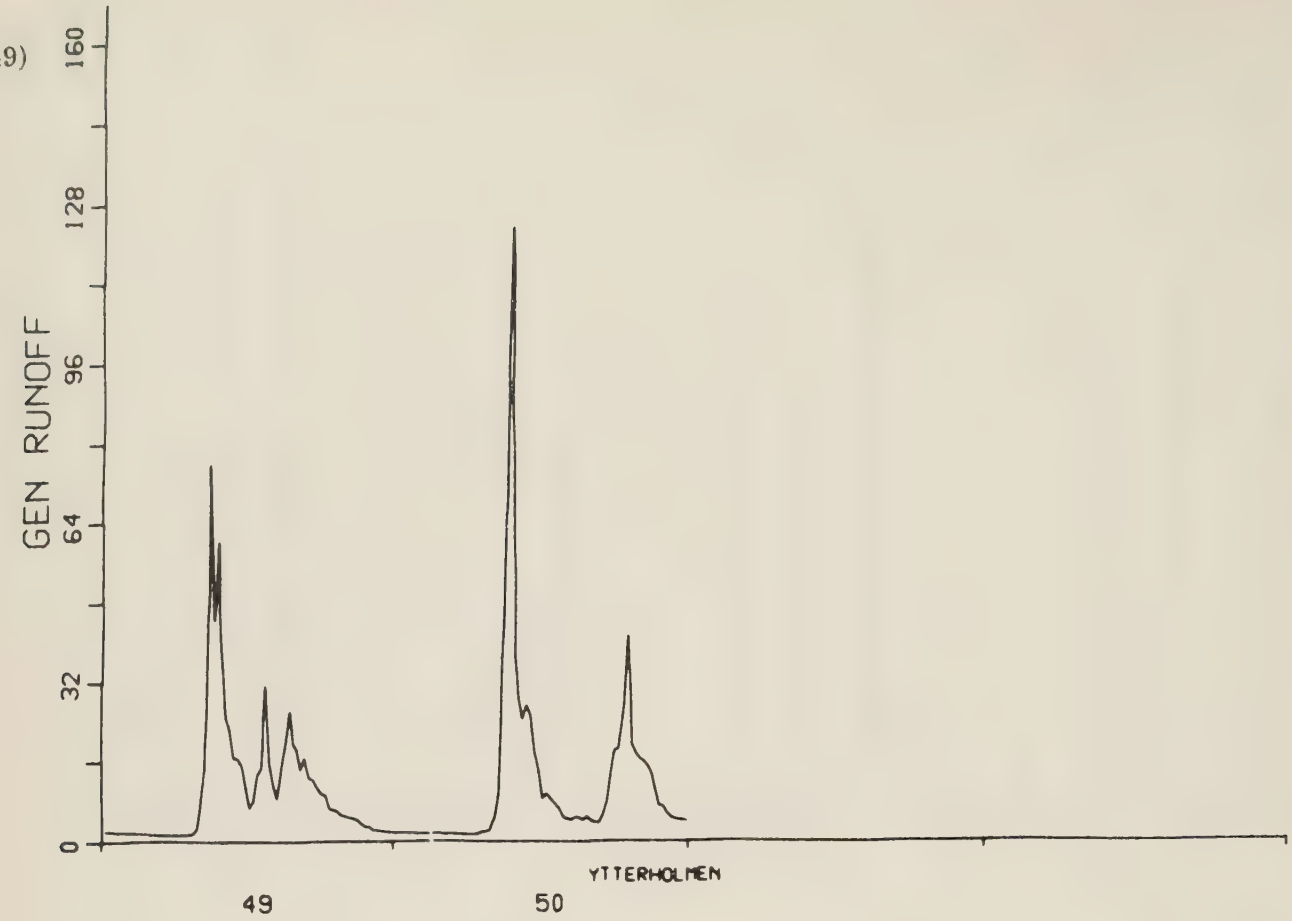
160
YTTERHOLMEN
45
46
47
48

Fig 12. cont.



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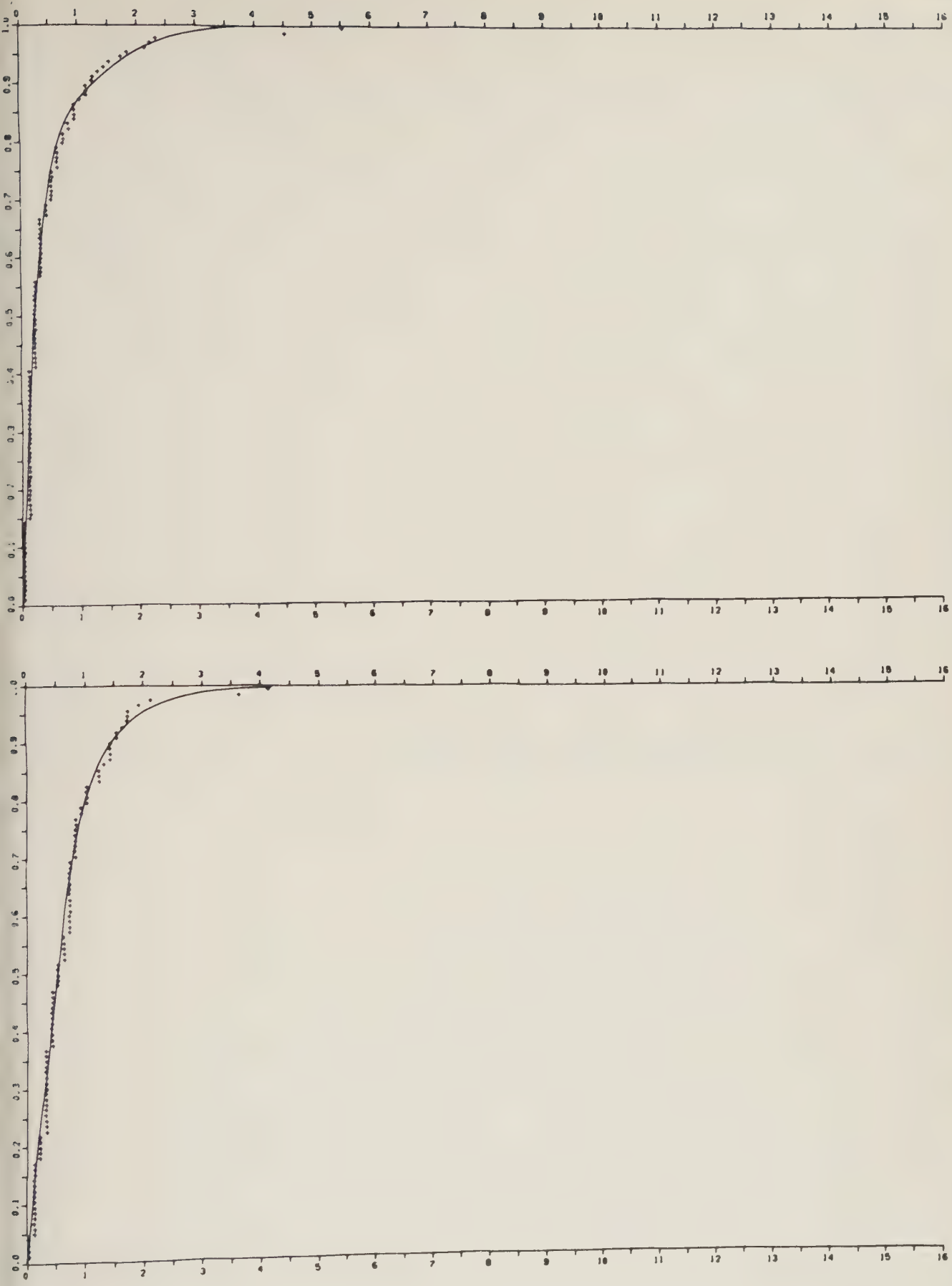


Fig 13. Conditional distributions for Korsvattnet, period 1 class 1 above, period 1 class 2 below.
Unit: VE F(X i) (solid line), $P(X^m)$ (+), see 5.2.

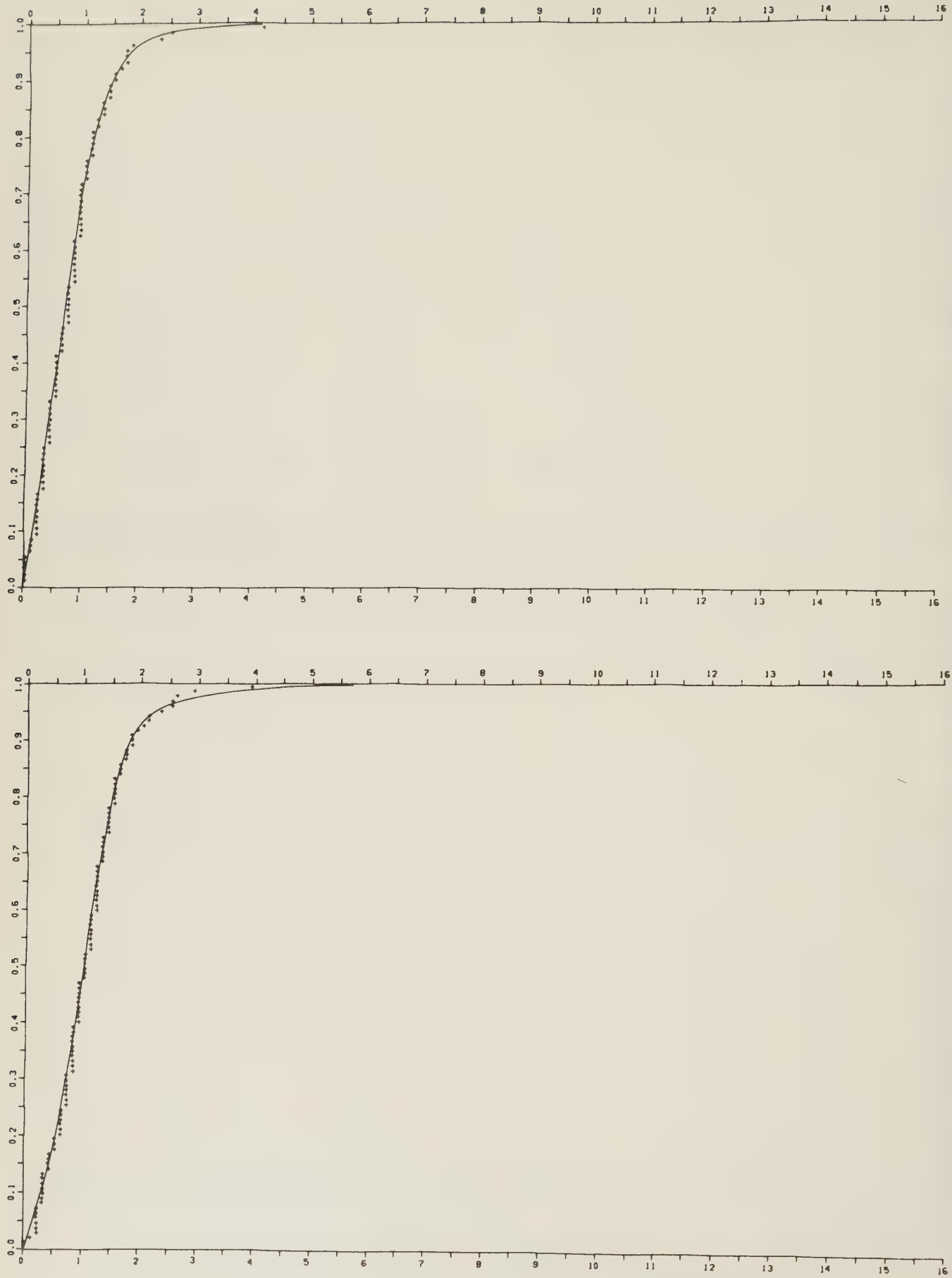
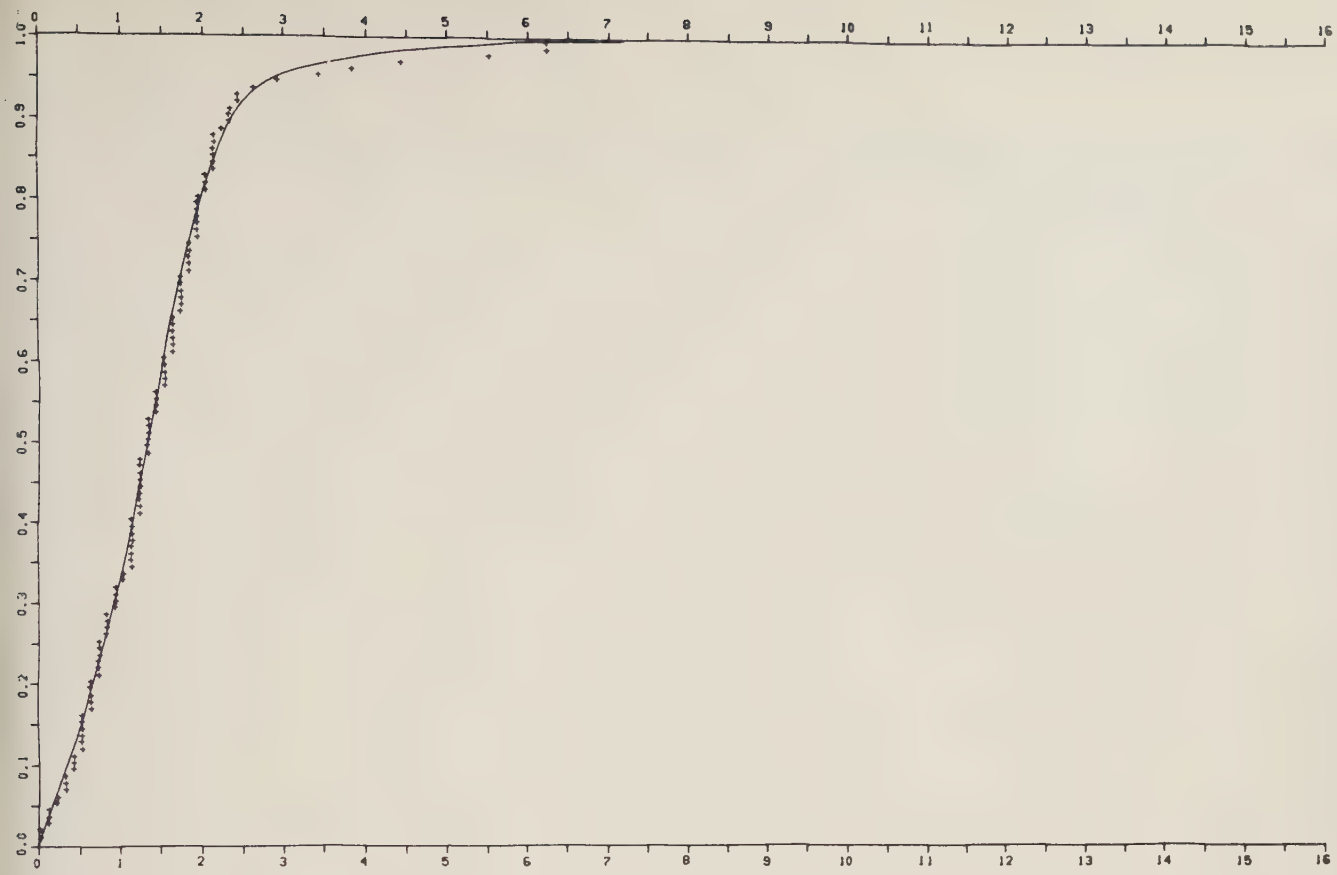


Fig 13. Conditional distributions for Korsvattnet, period 1 class 3 above, period 1 class 4 below.
Unit: VE



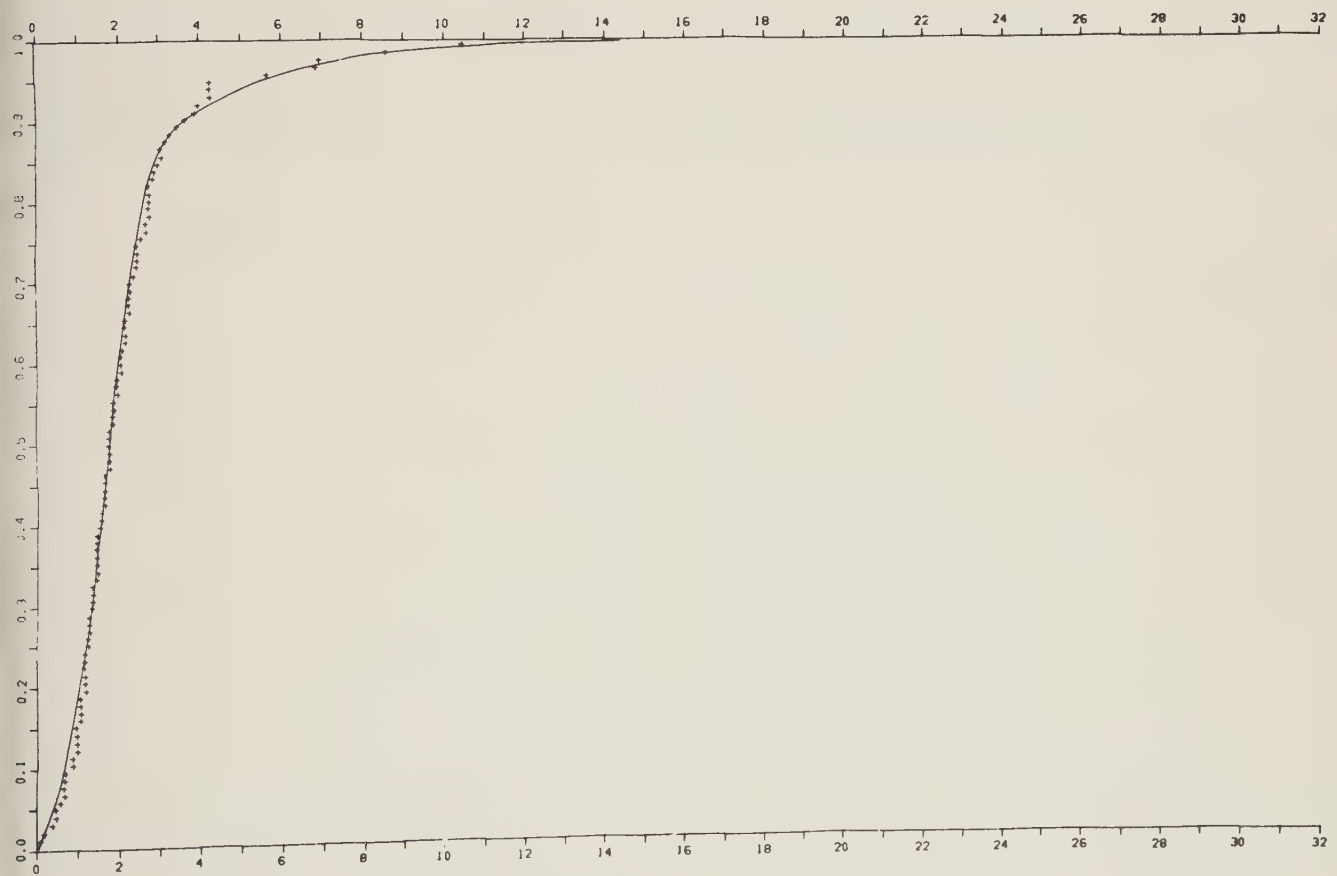


Fig 13. Conditional distributions for Korsvattnet, period 1 class 5 above, period 1 class 6 below.
Unit: VE

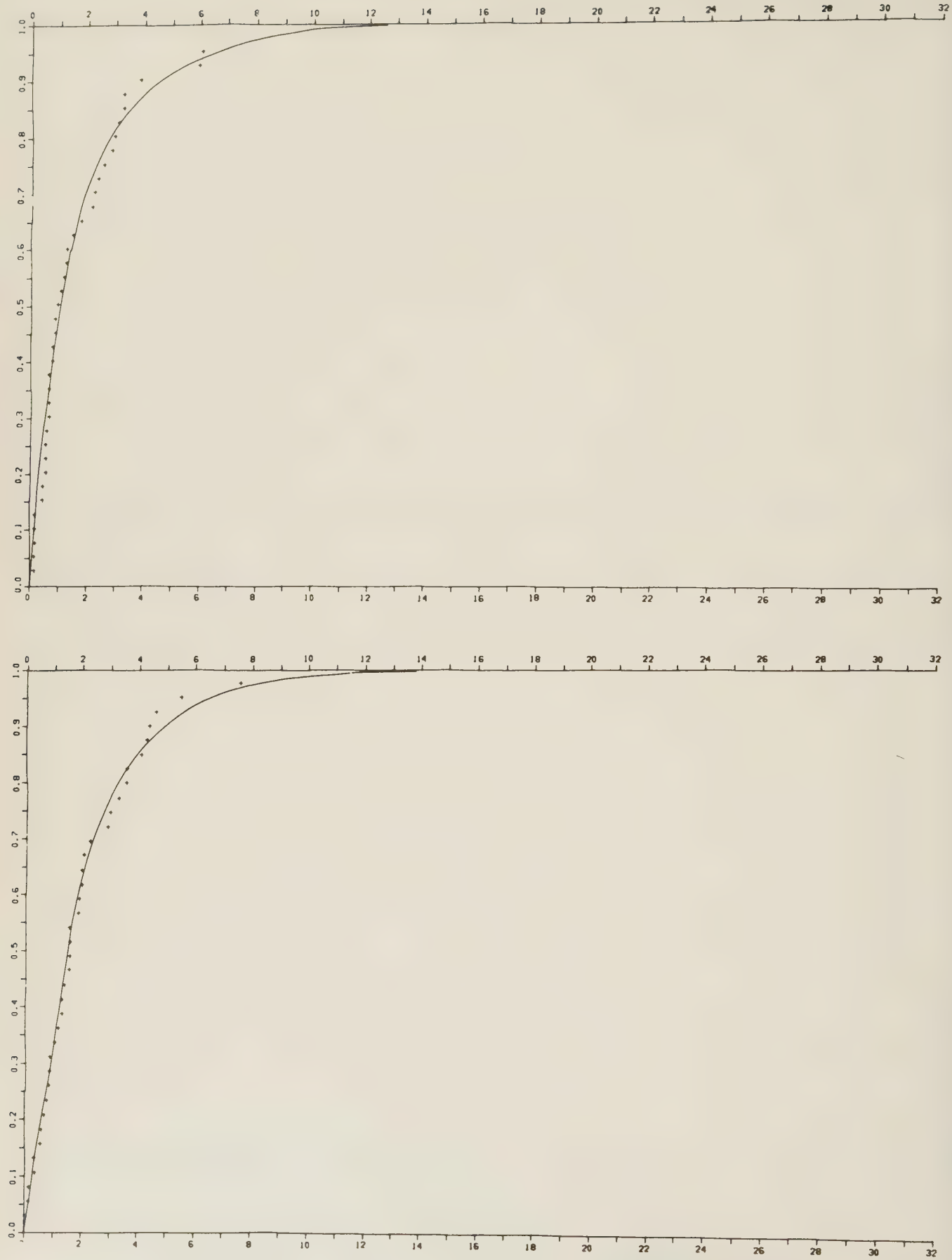


Fig 13. Conditional distributions for Korsvattnet, period 3 class 1 above, period 3 class 2 below.
Unit: VE

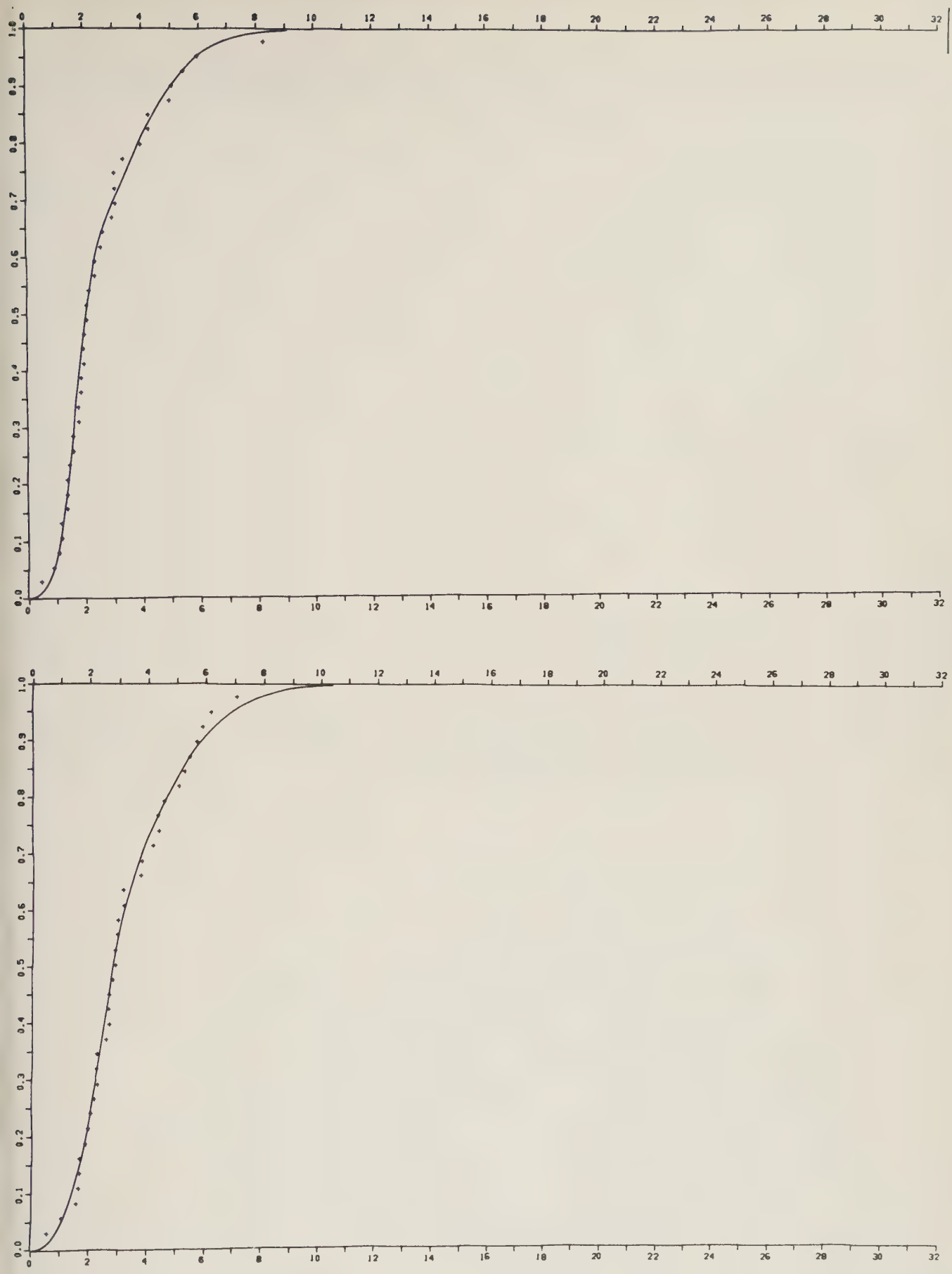


Fig 13. Conditional distributions for Korsvattnet, period 3 class 3 above, period 3 class 4 below.
Unit: VE

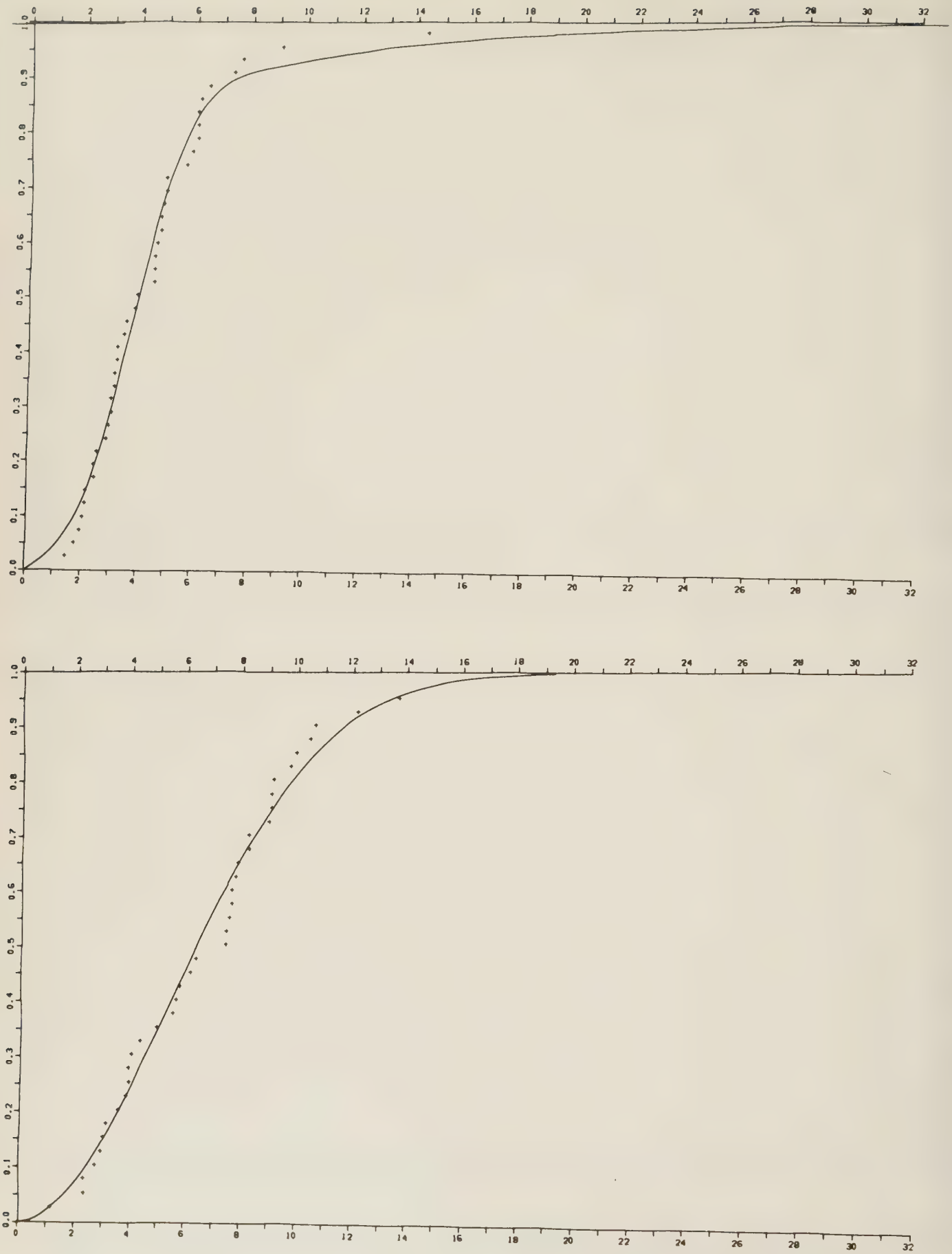


Fig 13. Conditional distributions for Korsvattnet, period 3 class 5 above, period 3 class 6 below.
Unit: VE

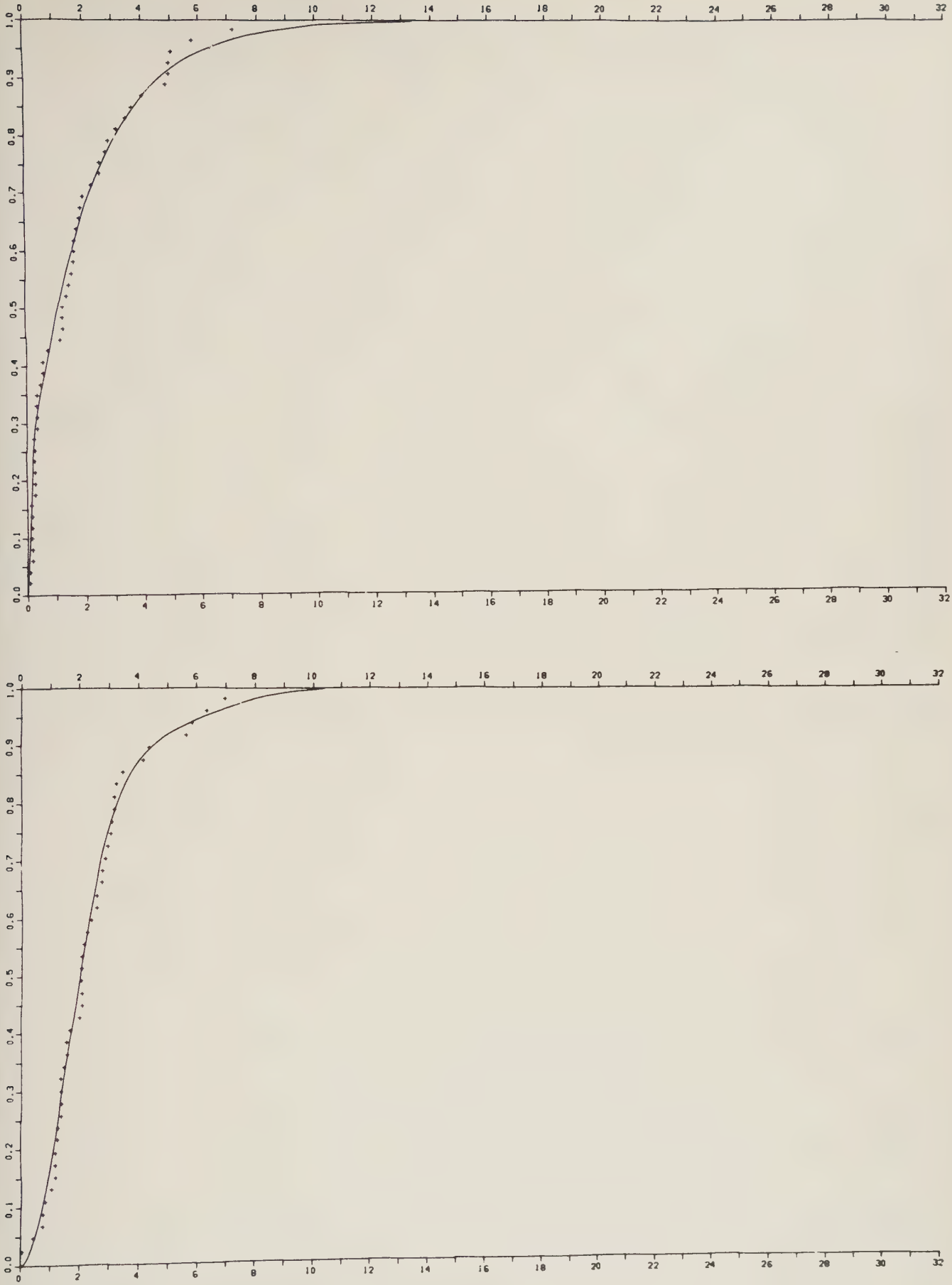


Fig 13 Conditional distributions for Korsvattnet, period 4 class 1 above, period 4 class 2 below.
Unit: VE

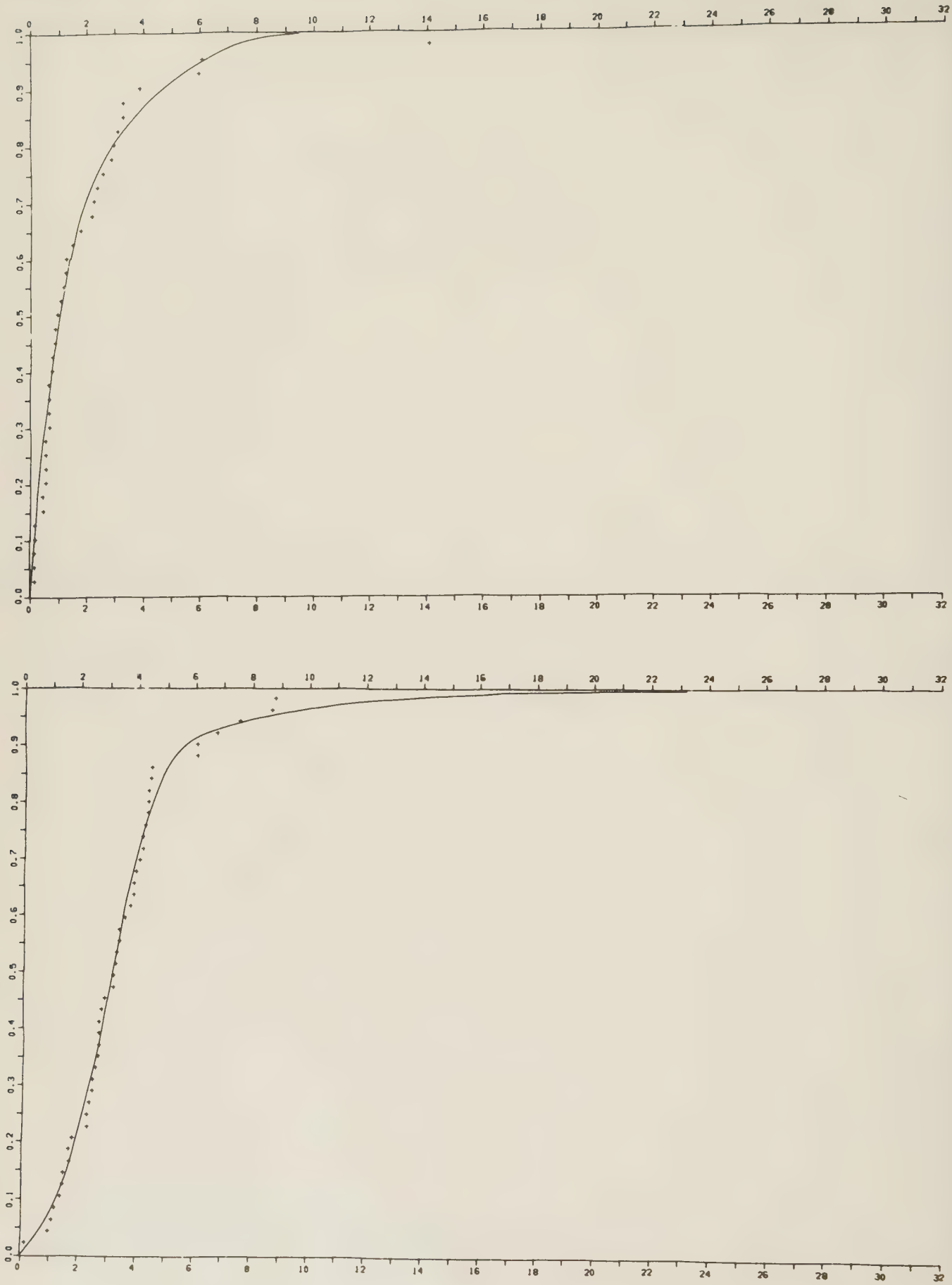


Fig 13. Conditional distributions for Korsvattnet, period 4 class 3 above, period 4 class 4 below.
Unit: VE

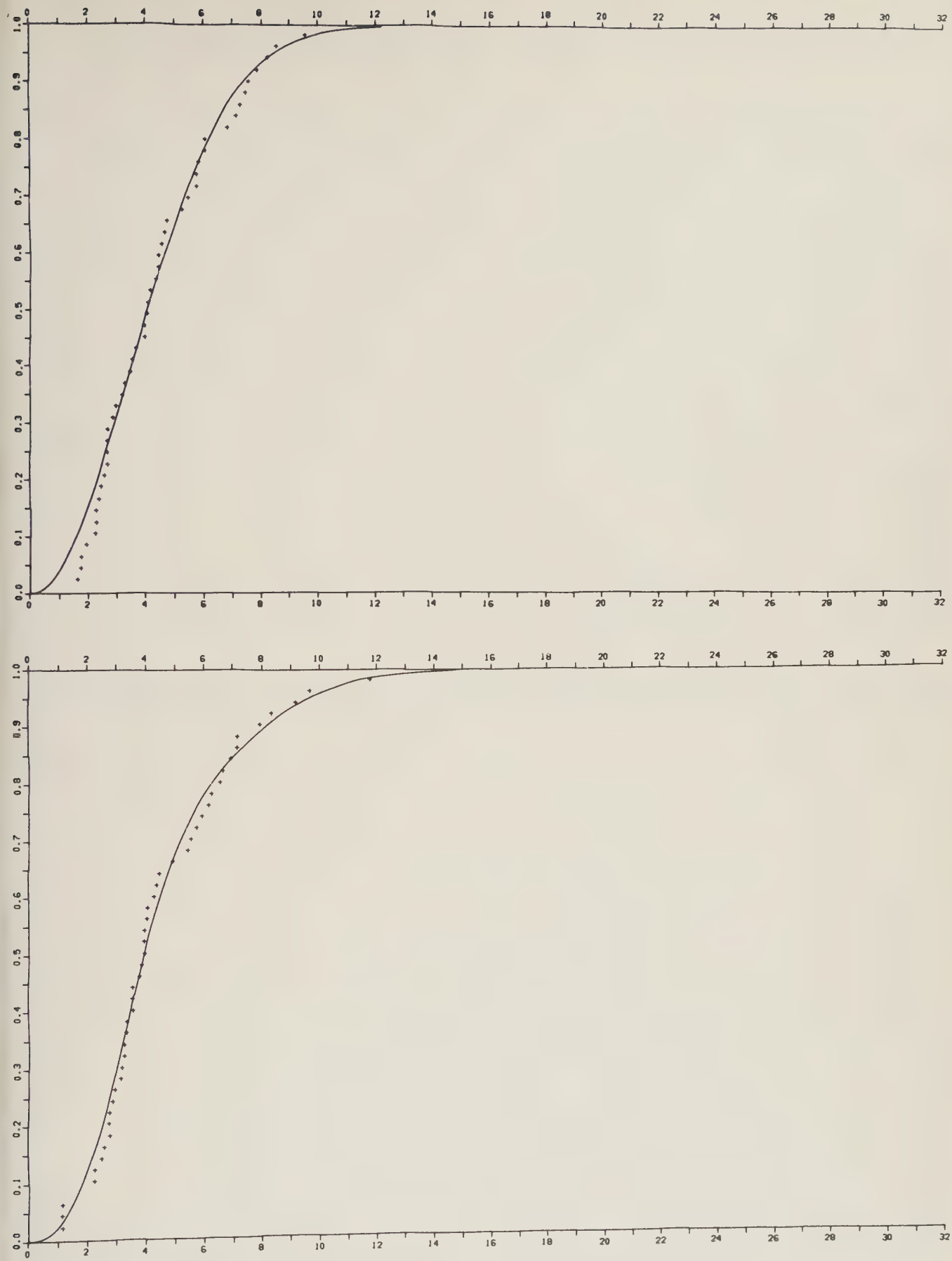


Fig 13 Conditional distributions for Korsvattnet, period 4 class 5 above, period 4 class 6 below.
Unit: VE

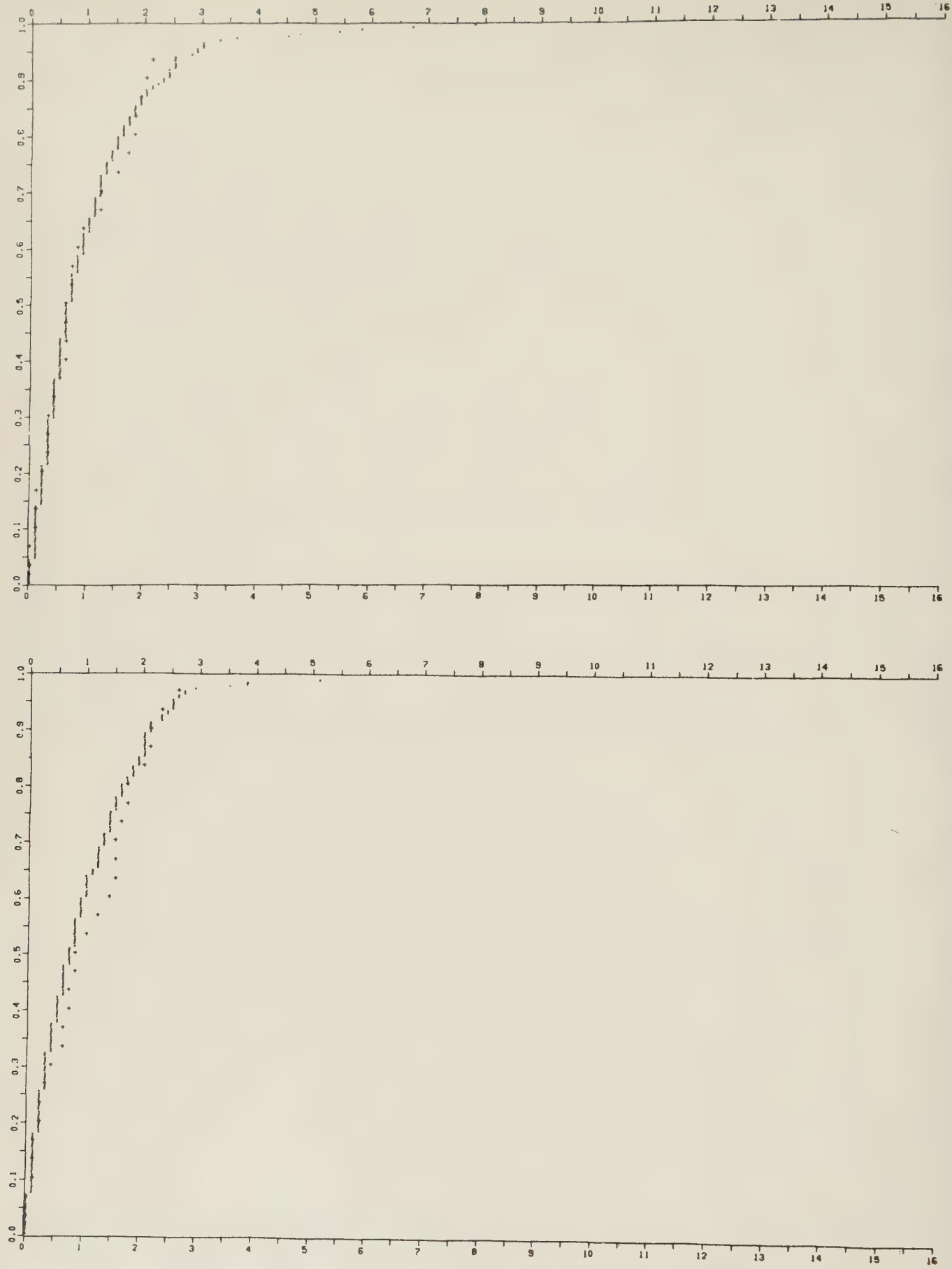
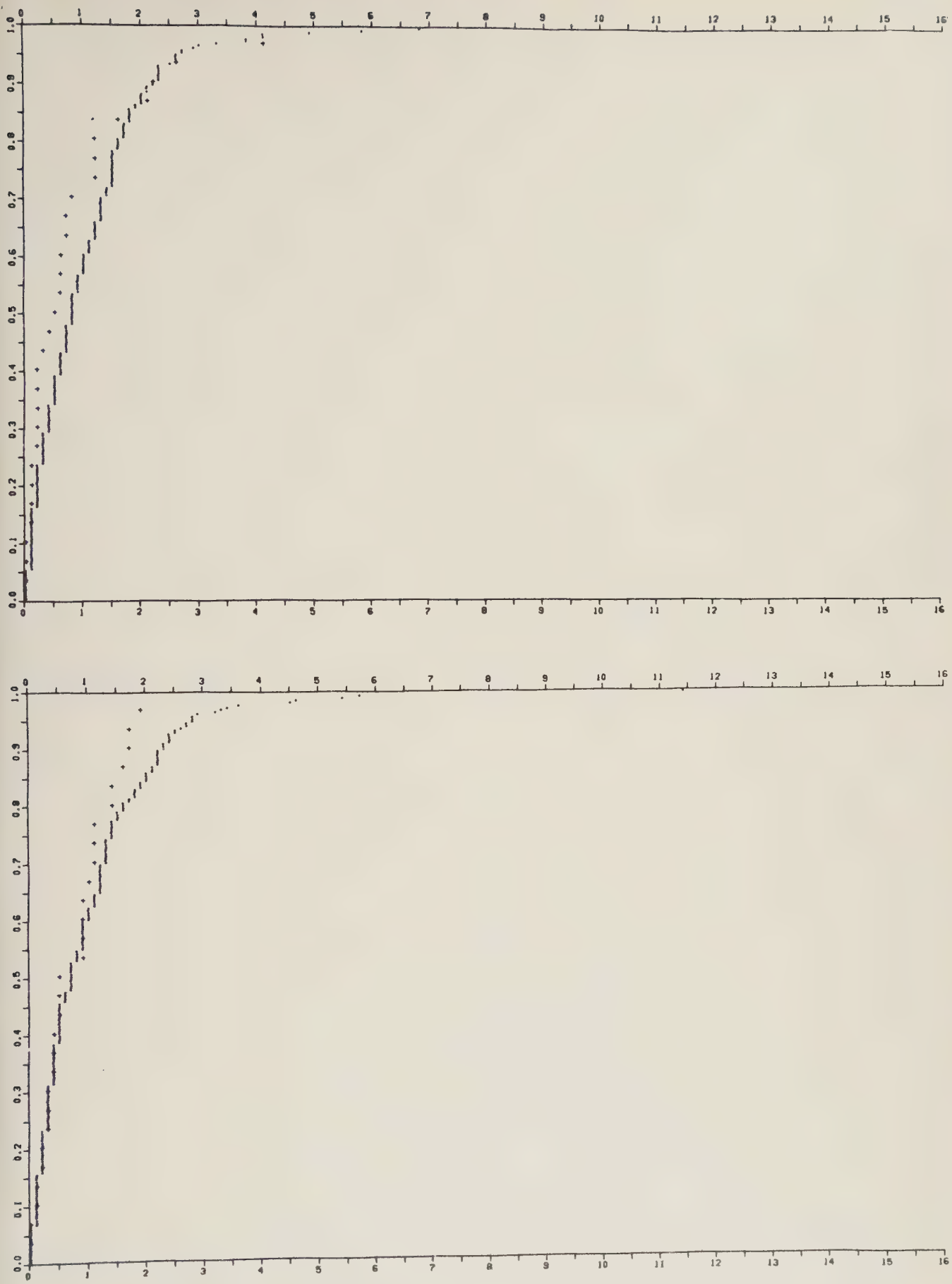


Fig 14 Unconditional distributions of weekly inflows for Korsvattnet.
Week 1 above, week 3 below.
Unit: VE Historic series (+), generated series (·).



g 14 Unconditional distributions of weekly inflows for Korsvattnet.
Week 5 above, week 7 below.
Unit: VE

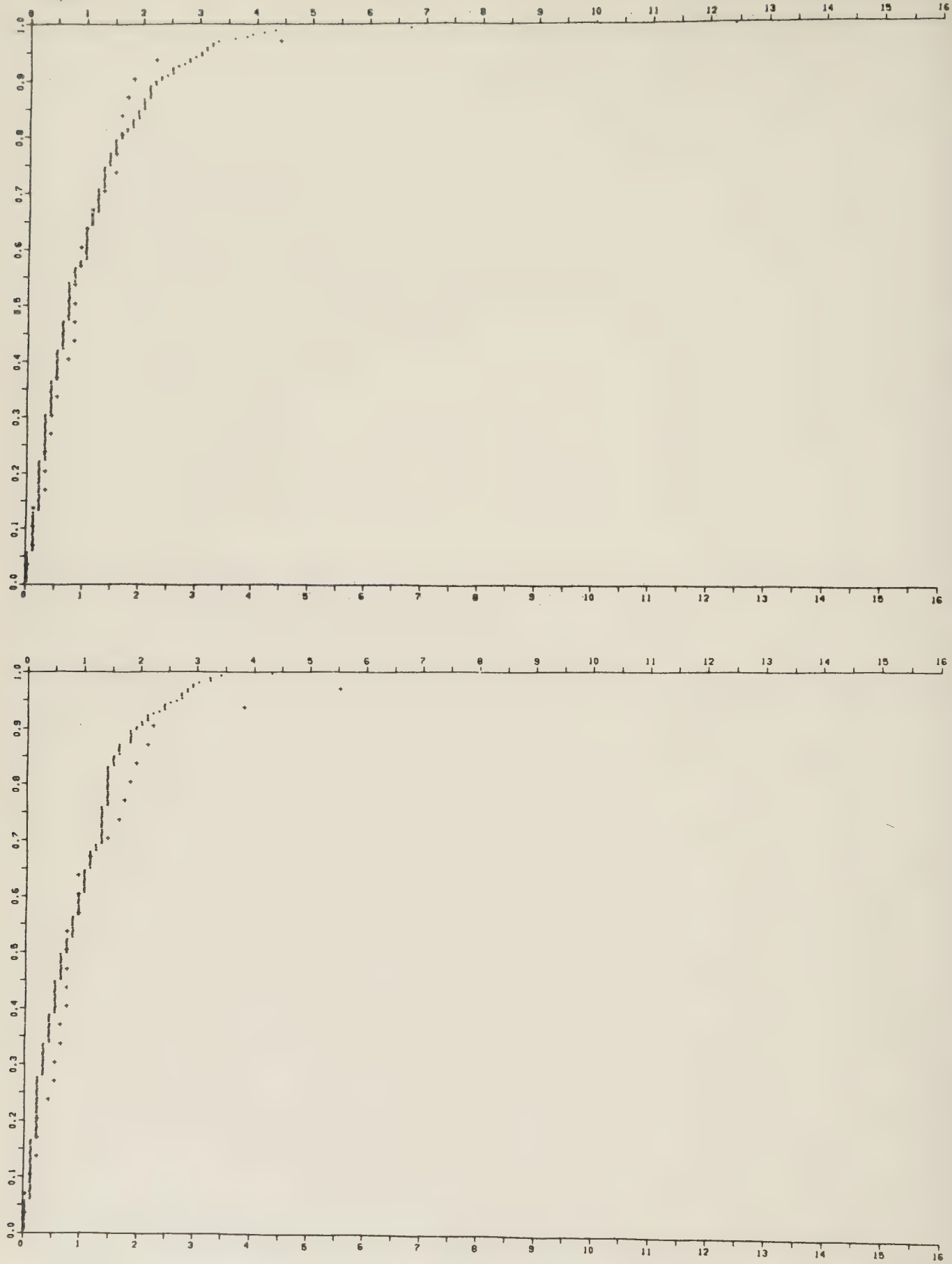


Fig 14 Unconditional distributions of weekly inflows for Korsvattnet.
Week 9 above, week 11 below.
Unit: VE

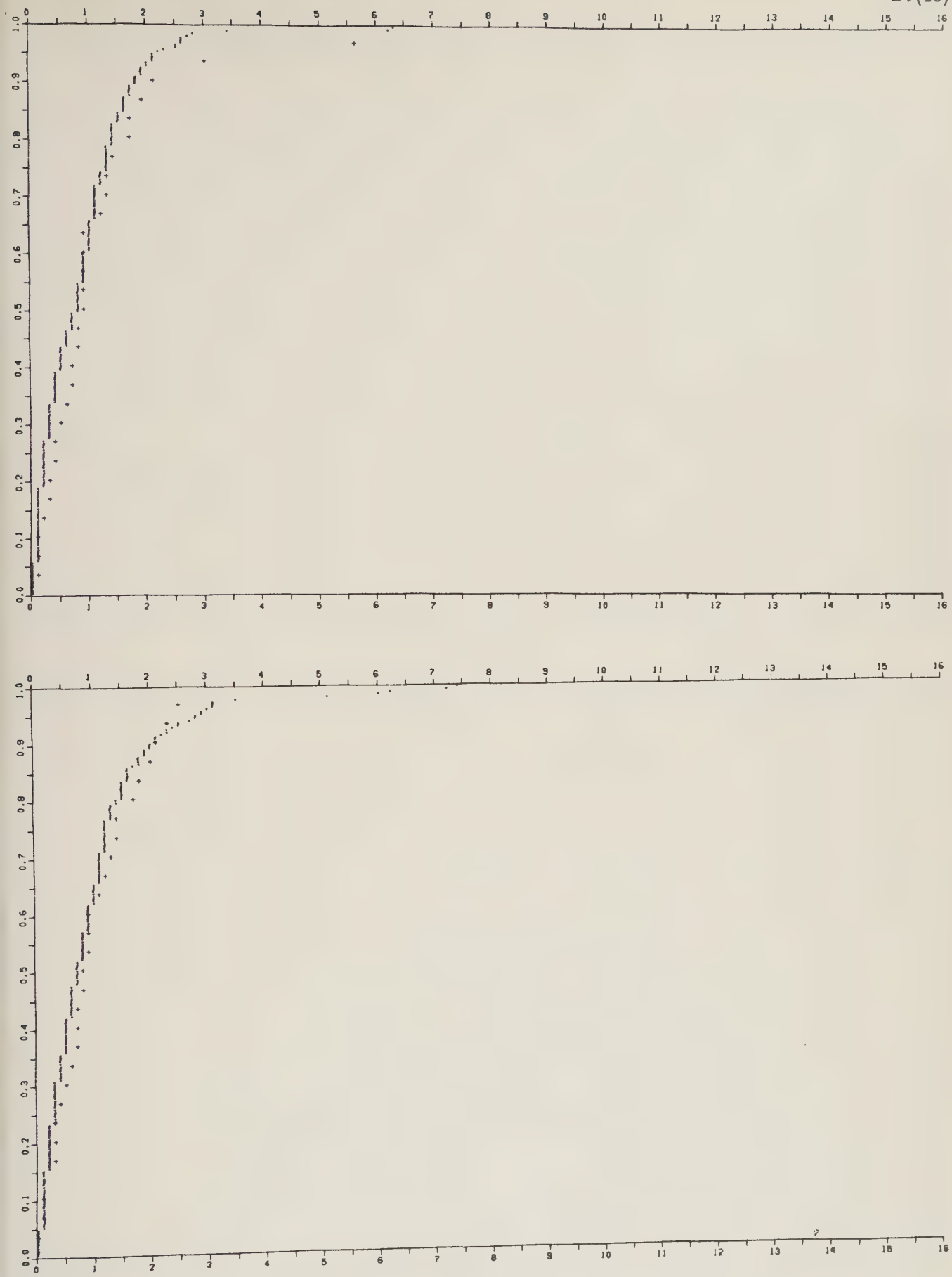


Fig 14 Unconditional distributions of weekly inflows for Korsvattnet.
Week 13 above, week 15 below.
Unit: VE

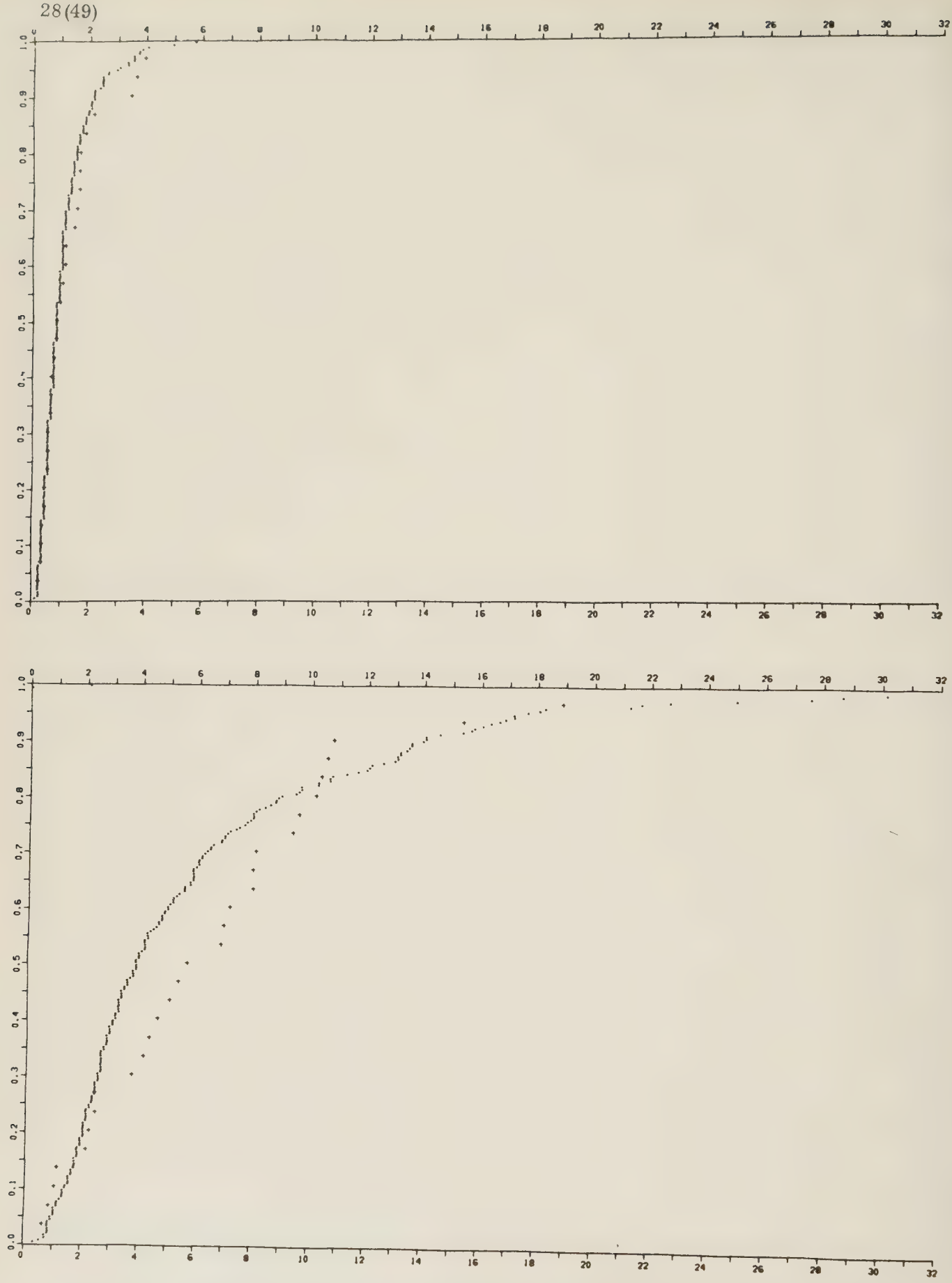


Fig 14 Unconditional distributions of weekly inflows for Korsvattnet.
Week 17 above, week 19 below.
Unit: VE

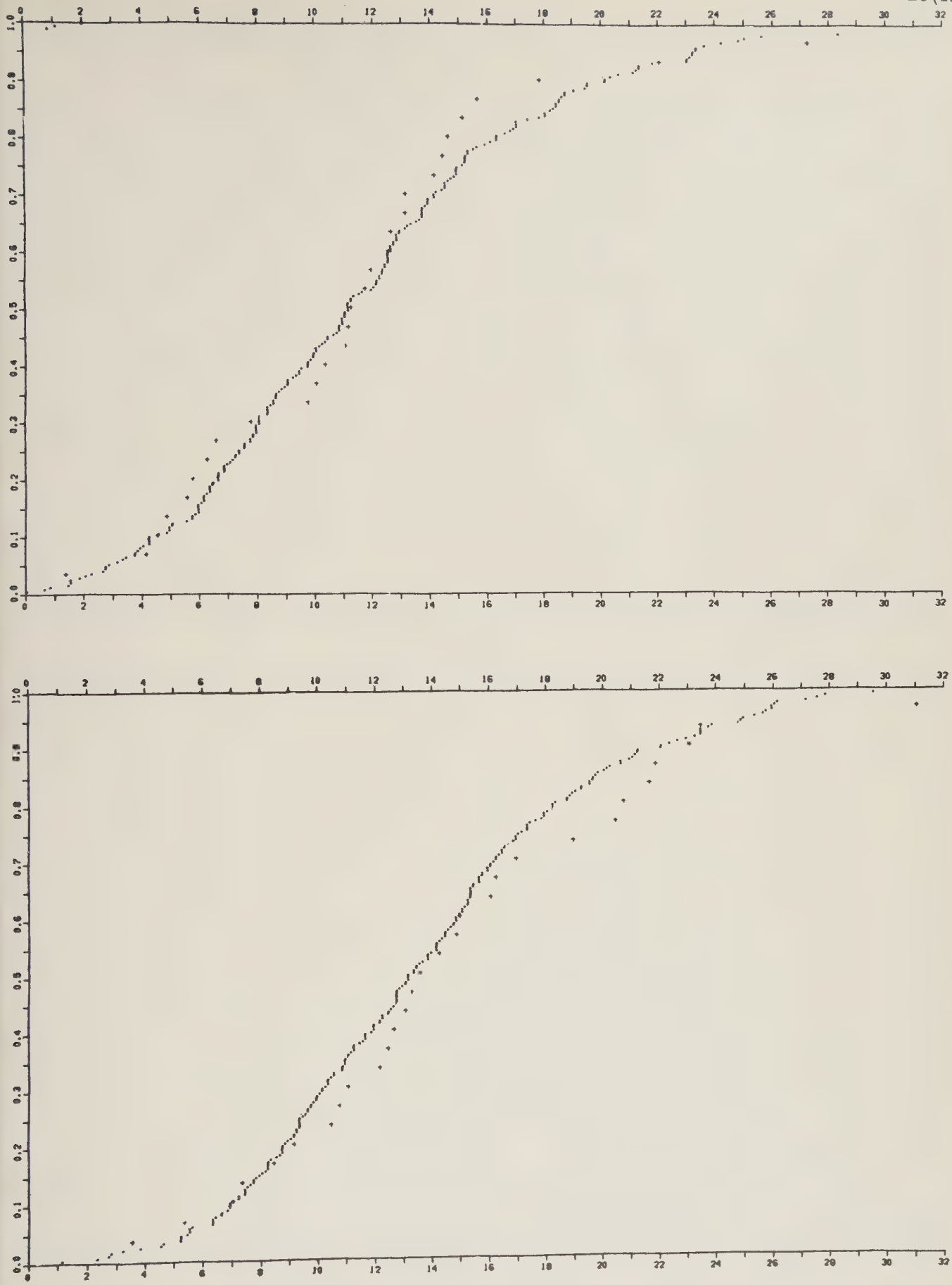


Fig 14 Unconditional distributions of weekly inflows for Korsvattnet.
Week 21 above, week 23 below.
Unit: VE

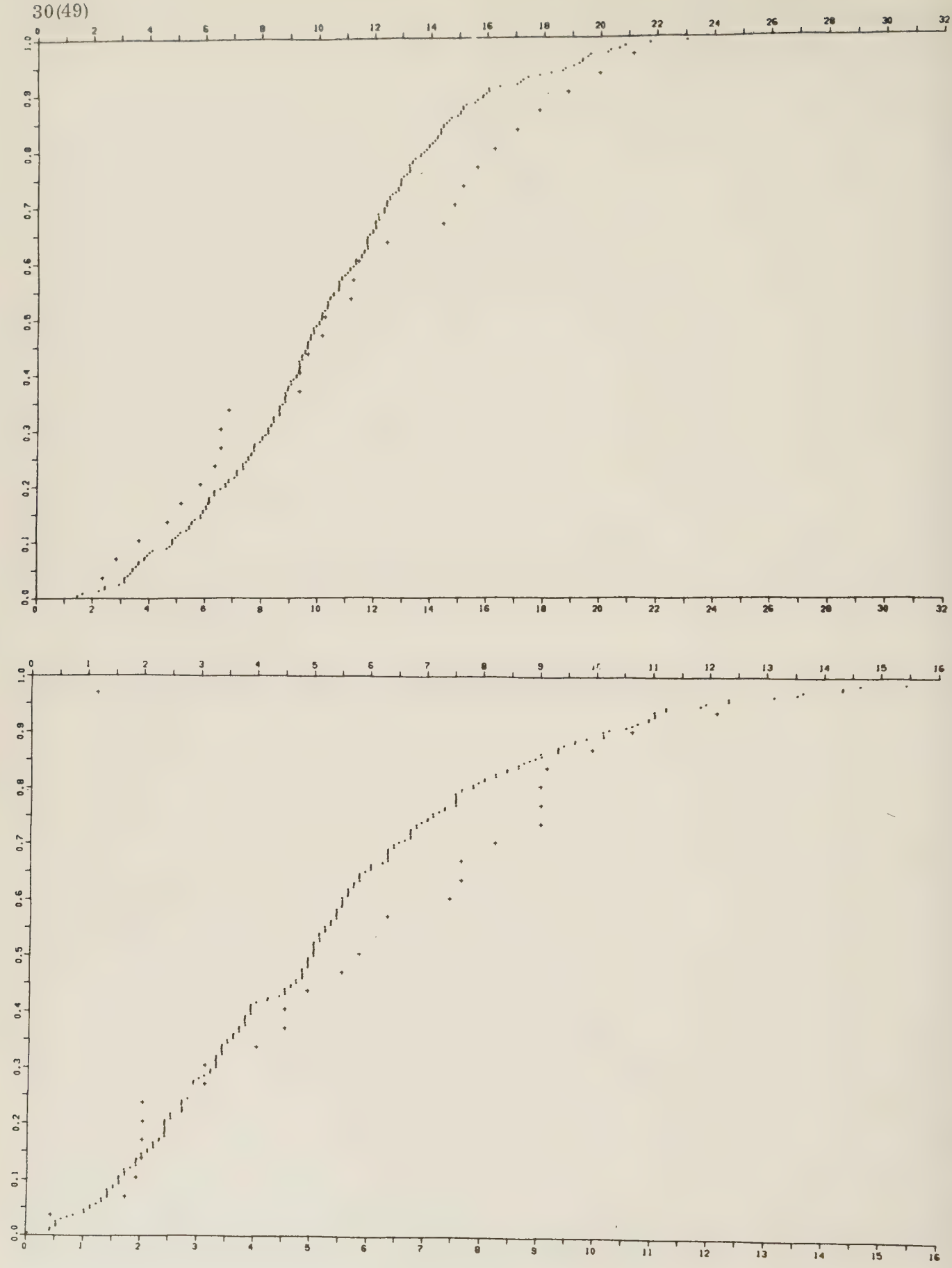


Fig 14 Unconditional distributions of weekly inflows for Korsvattnet.
Week 25 above, week 27 below.
Unit: VE

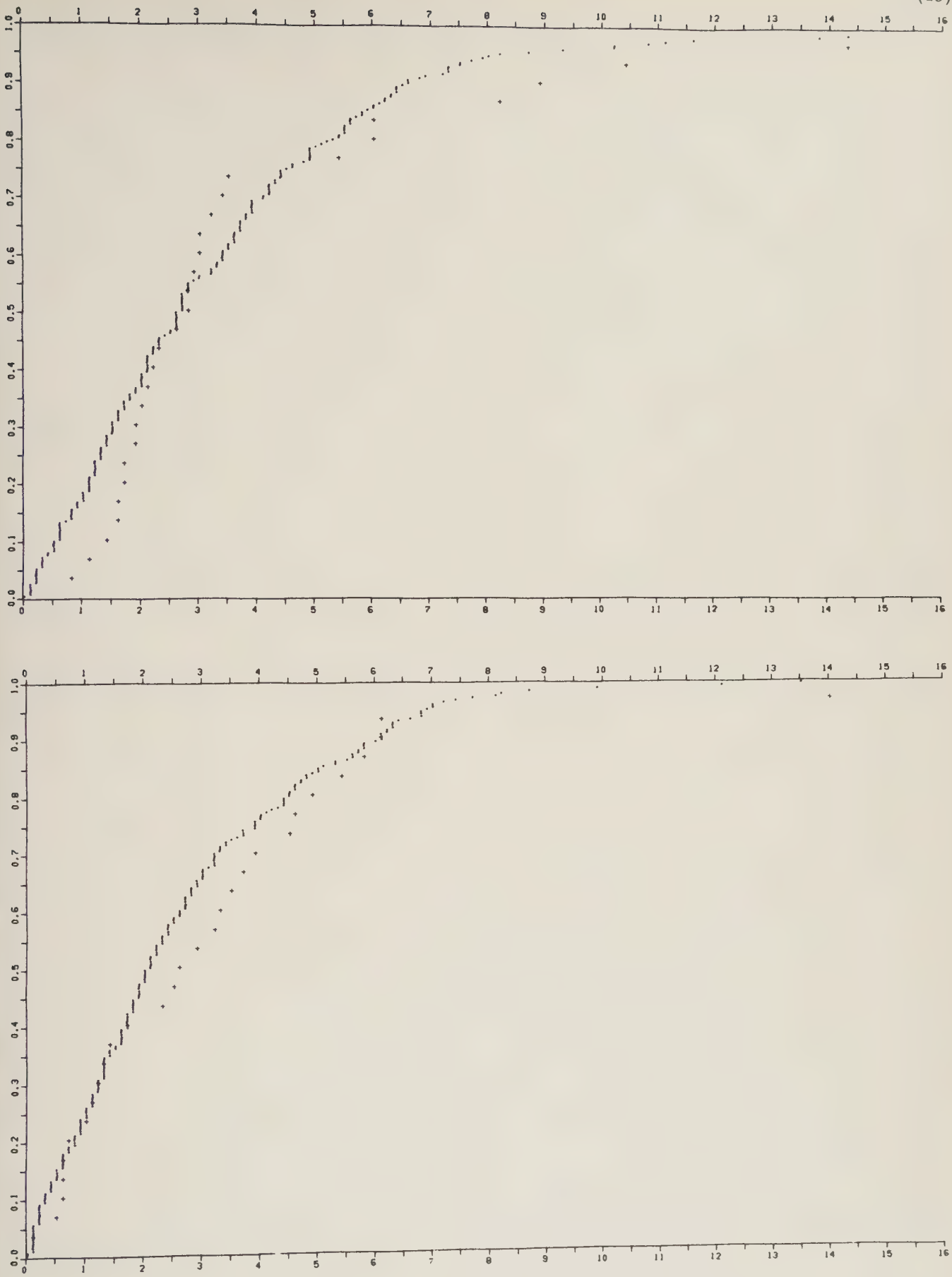


Fig 14 Unconditional distributions of weekly inflows for Korsvattnet. Week 29 above, week 31 below.
Unit: VE

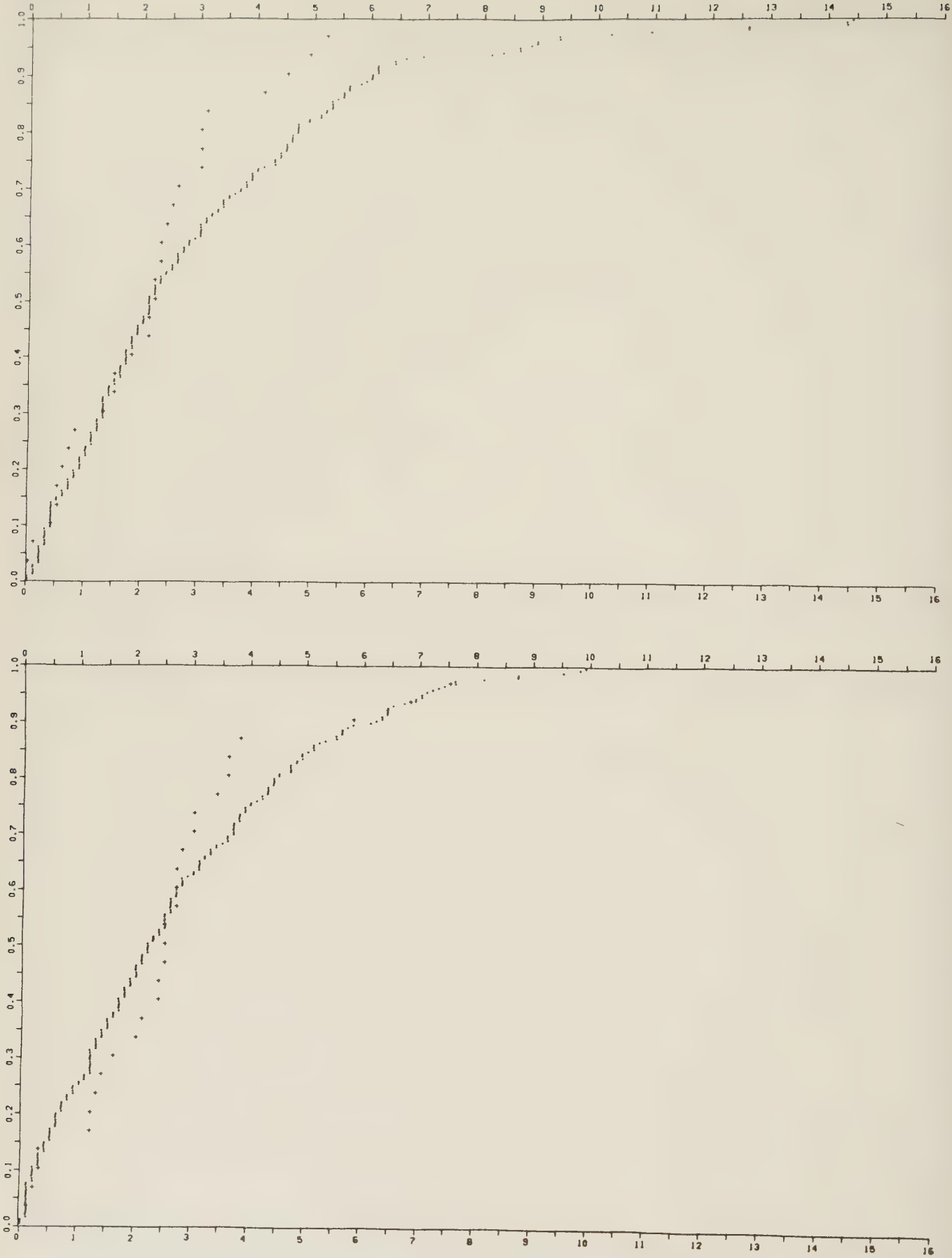


Fig 14 Unconditional distributions of weekly inflows for Korsvattnet.
Week 33 above, week 35 below.
Unit: VE

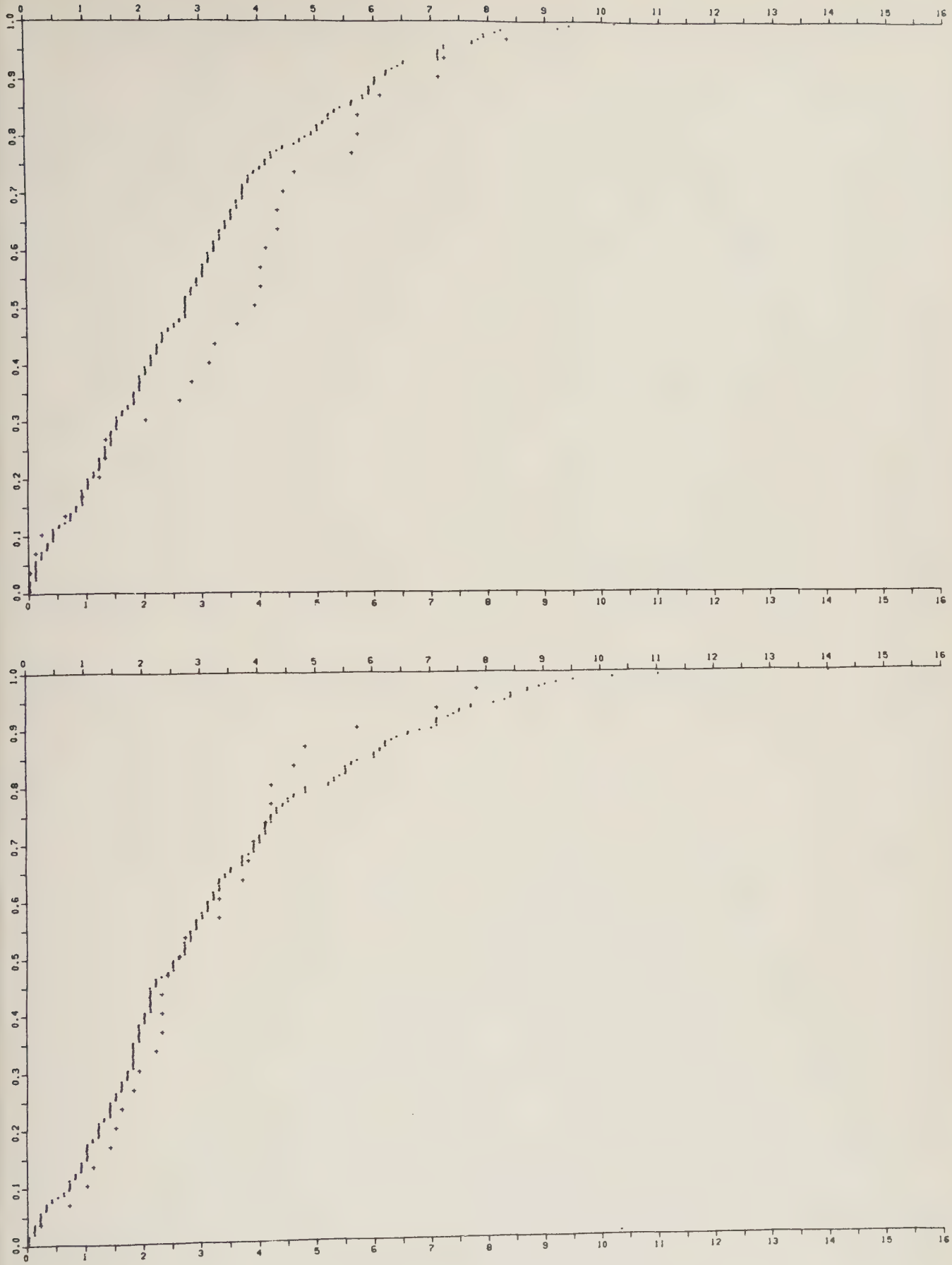


Fig 14 Unconditional distributions of weekly inflows for Korsvattnet.
Week 37 above, week 39 below.
Unit: VE

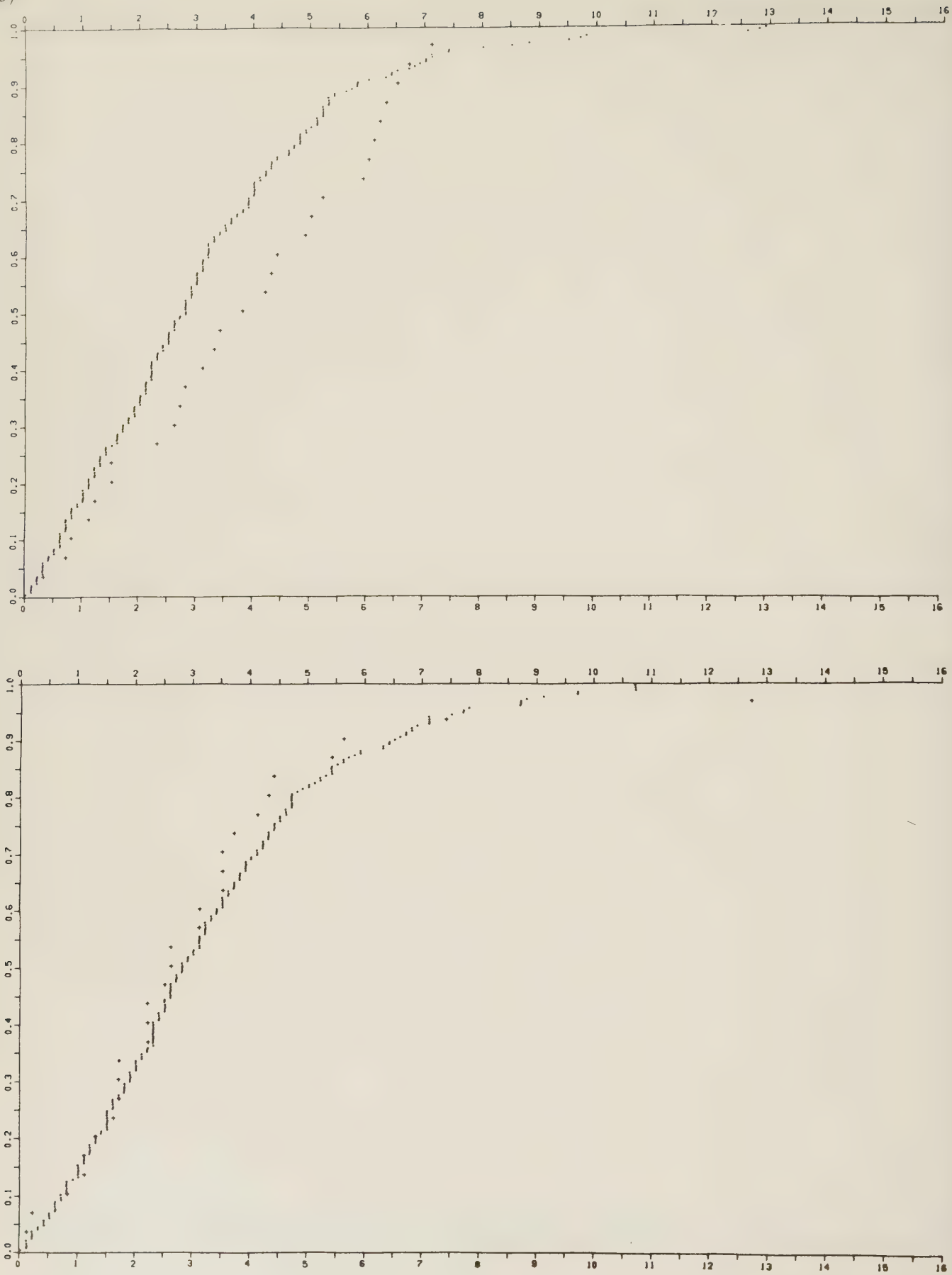


Fig 14 Unconditional distributions of weekly inflows for Korsvattnet.
Week 41 above, week 43 below.
Unit: VE

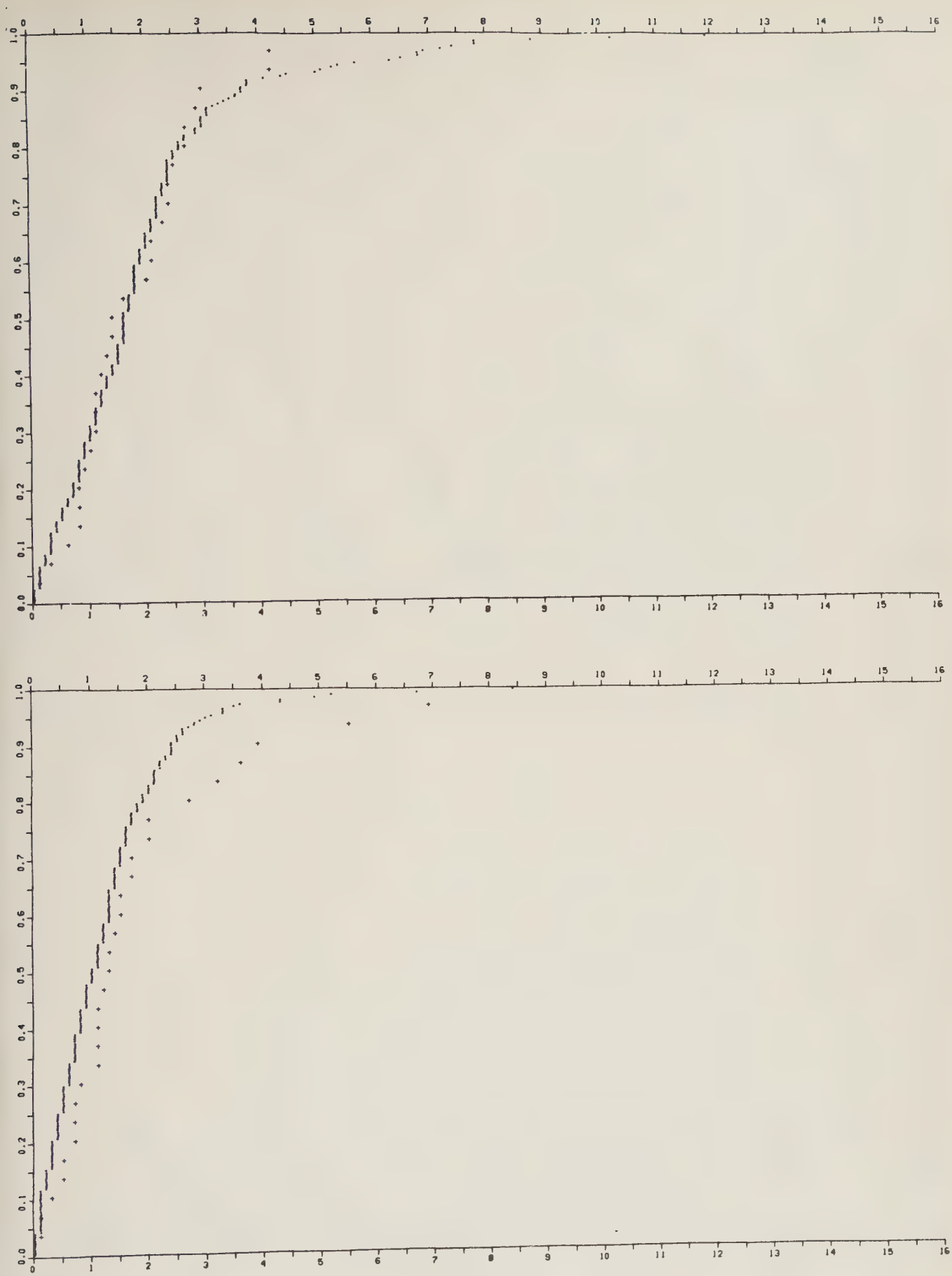


Fig 14 Unconditional distributions of weekly inflows for Korsvattnet.
Week 45 above, week 47 below.
Unit: VE

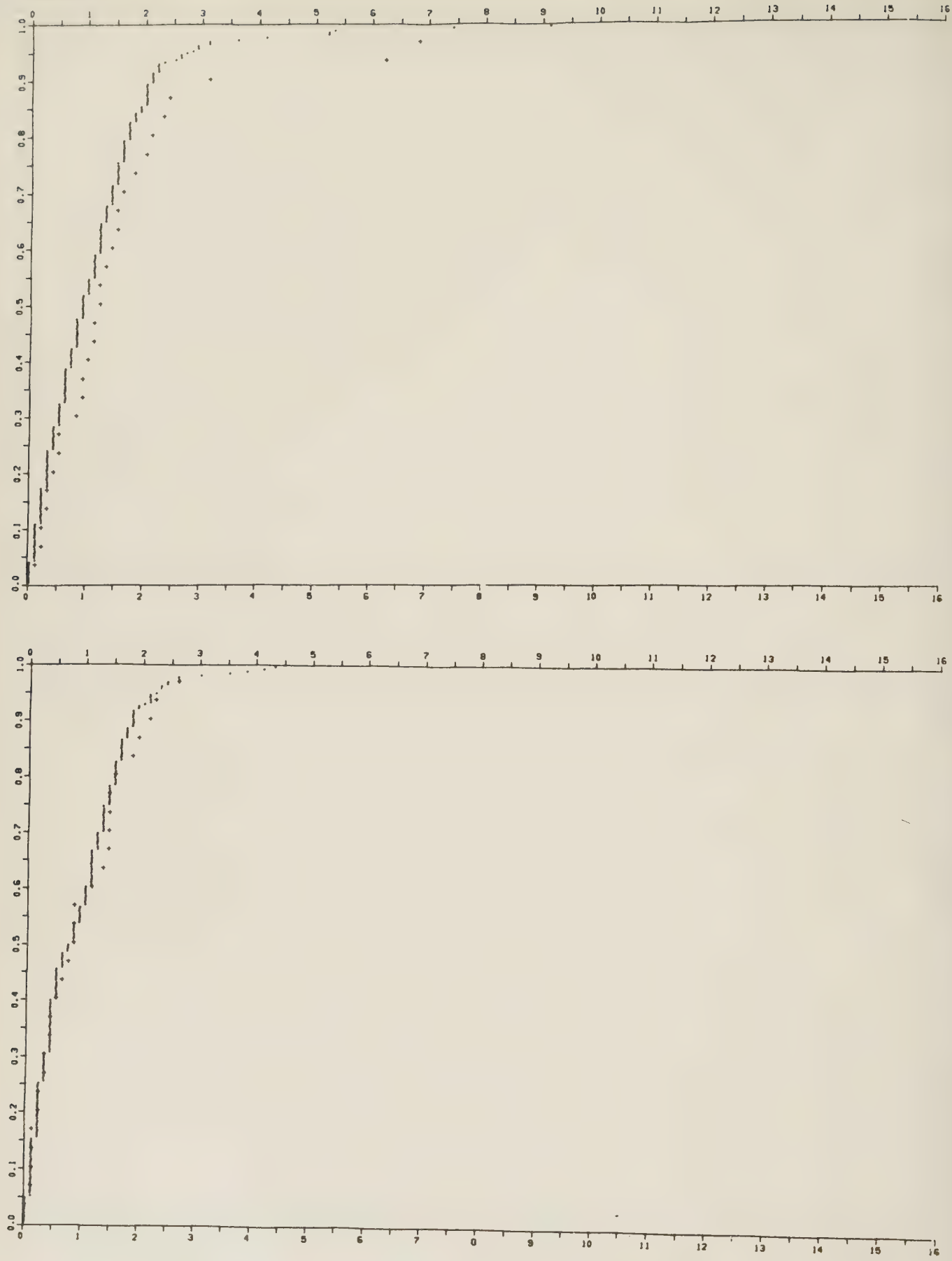


Fig 14 Unconditional distributions of weekly inflows for Korsvattnet.
Week 49 above, week 51 below.
Unit: VE

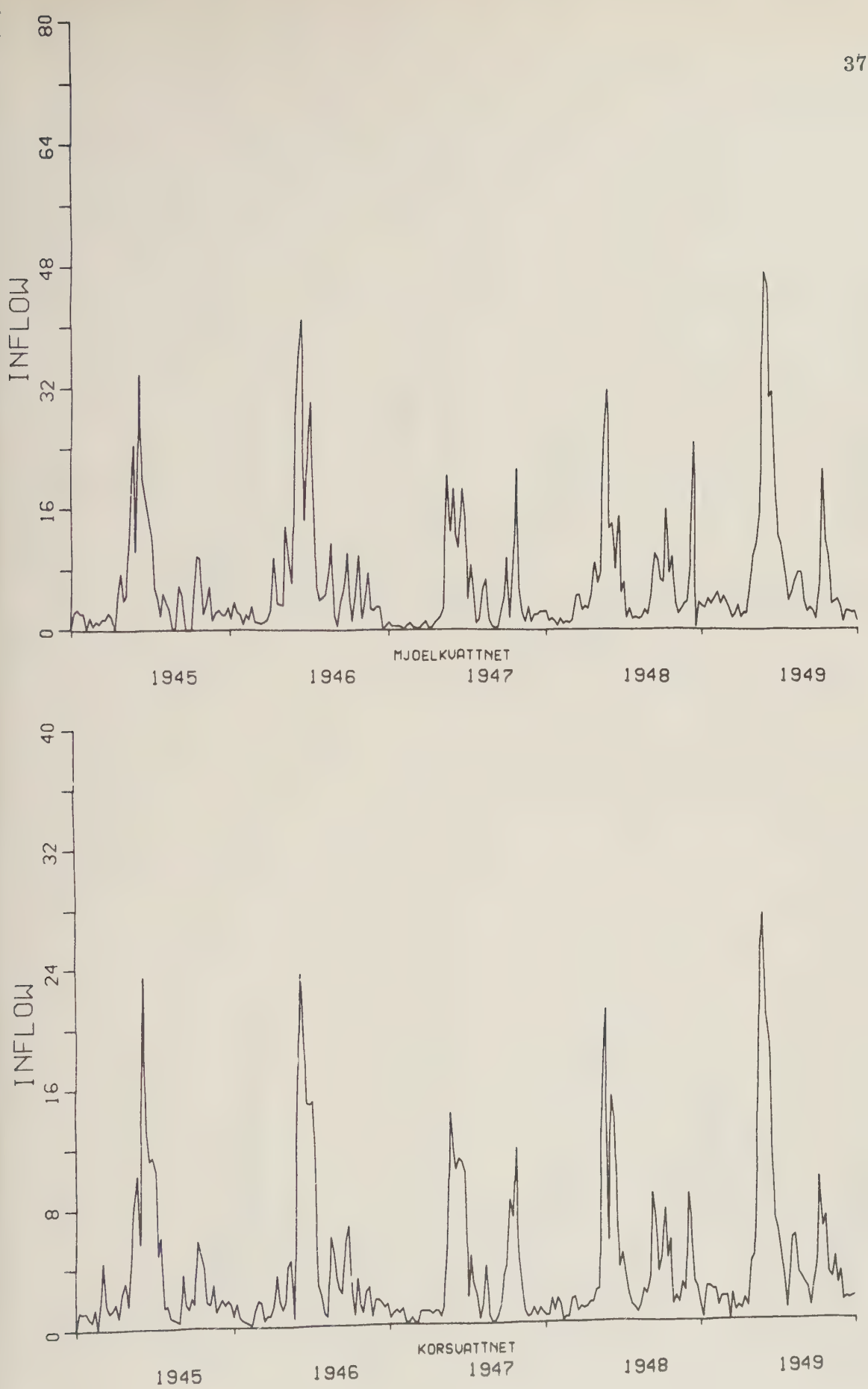
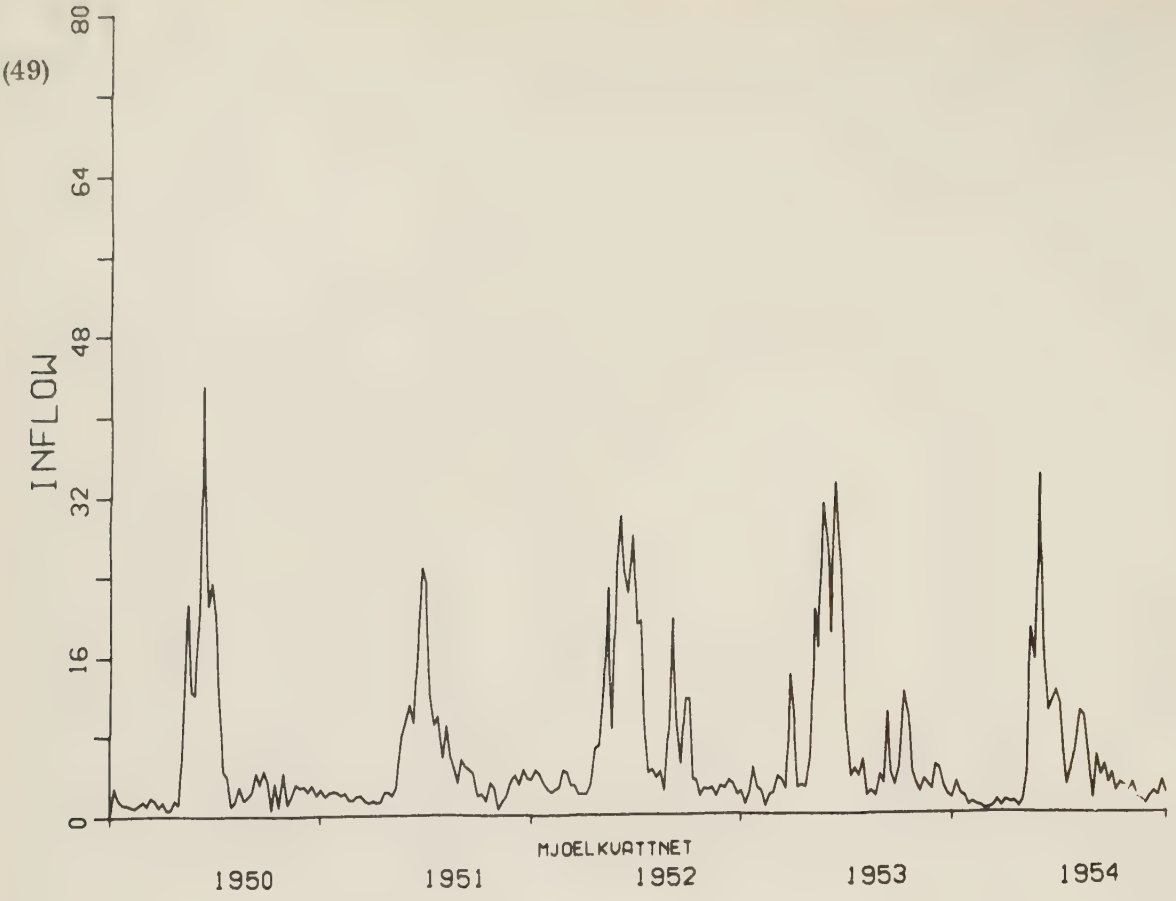


Fig 15 Historical series of weekly inflow to Stora Mjölkvattnet and Korsvattnet.
Unit: VE


INFLOW
0
16
32
48
64
80
MJOELKVATTNET
1950
1951
1952
1953
1954

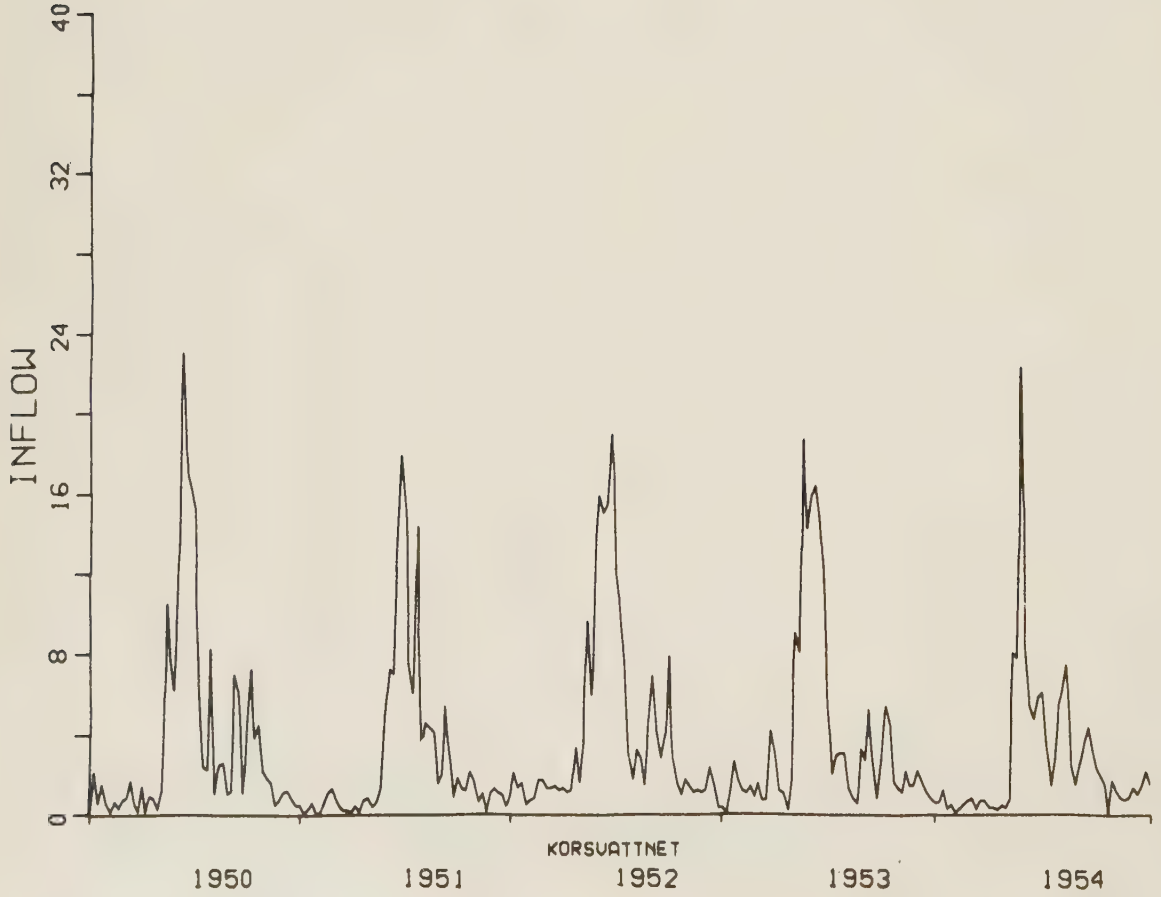
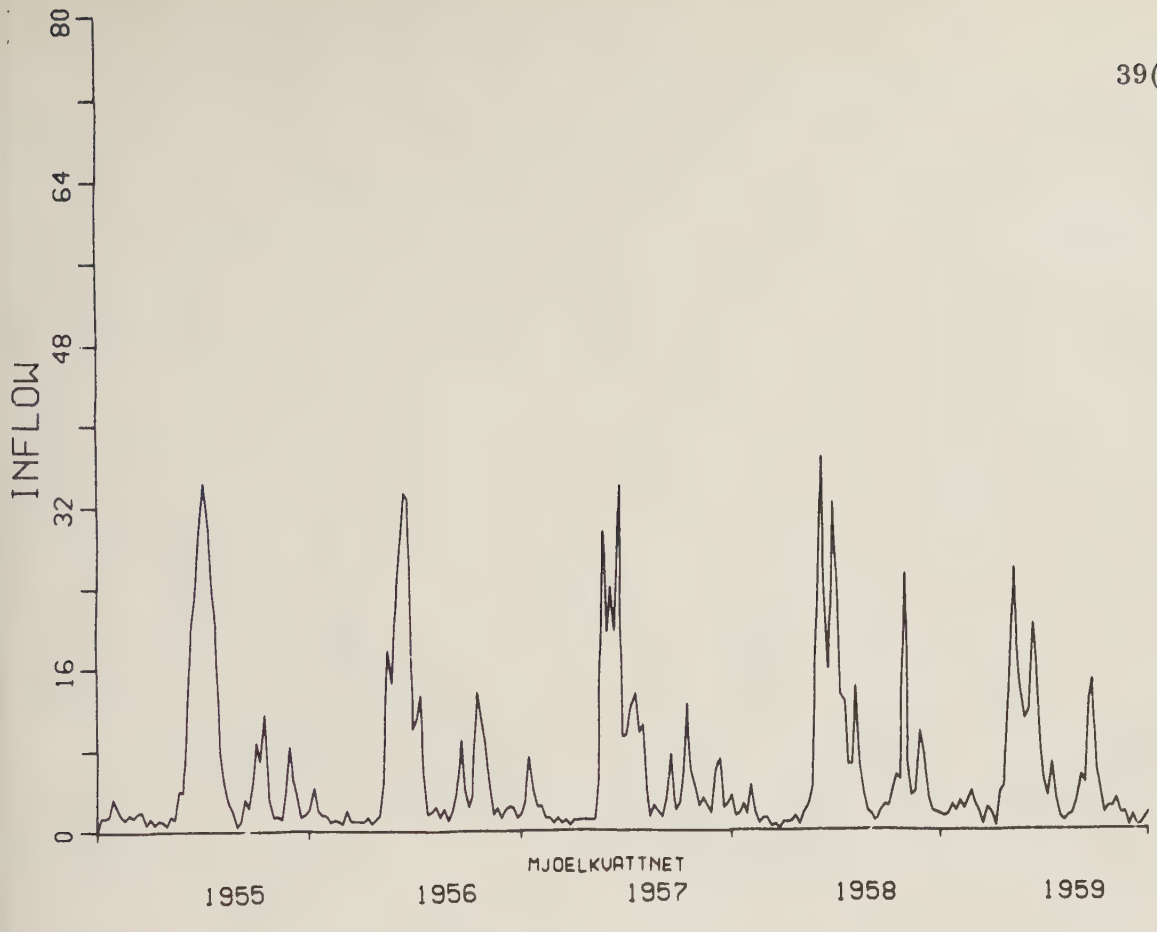

INFLOW
0
8
16
24
32
40
KORSVATTNET
1950
1951
1952
1953
1954

Fig 15 cont.


INFLOW
0
16
32
48
64
80
MJOELKVATTNET
1955
1956
1957
1958
1959

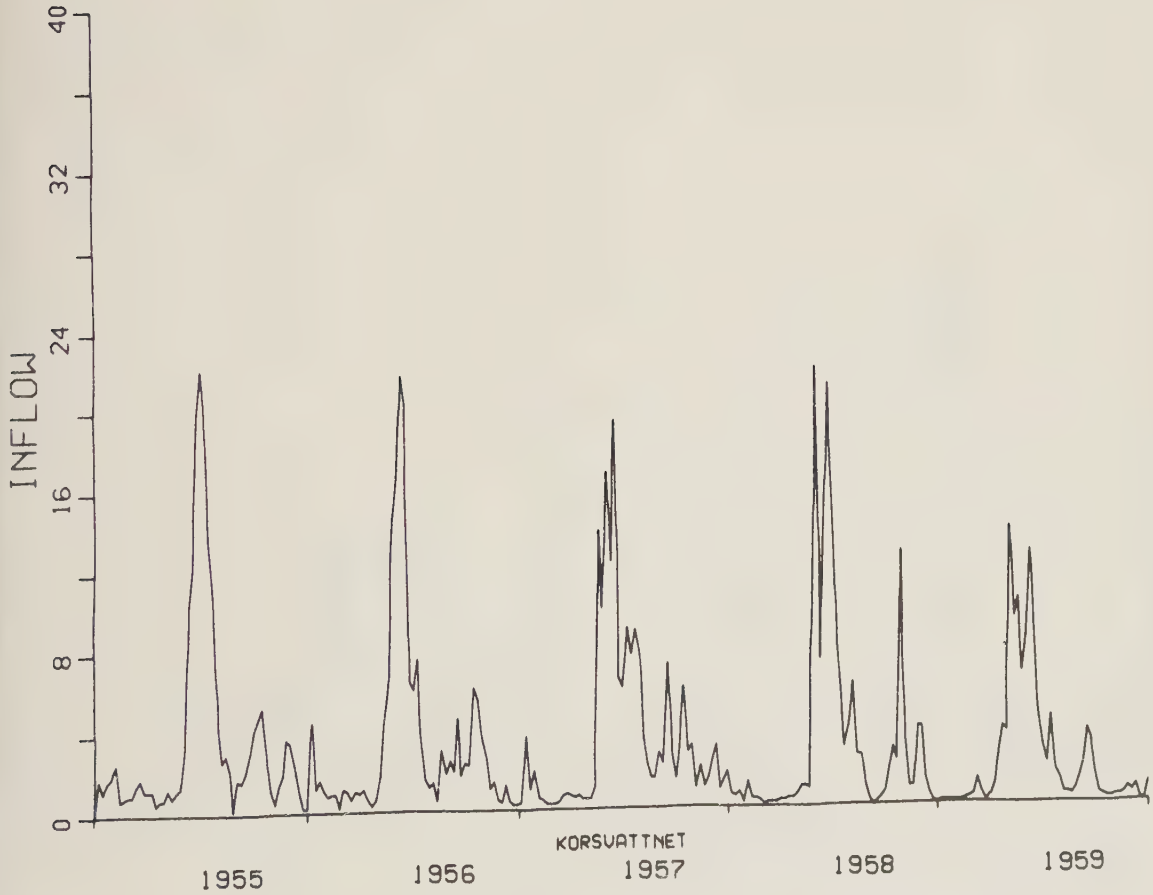
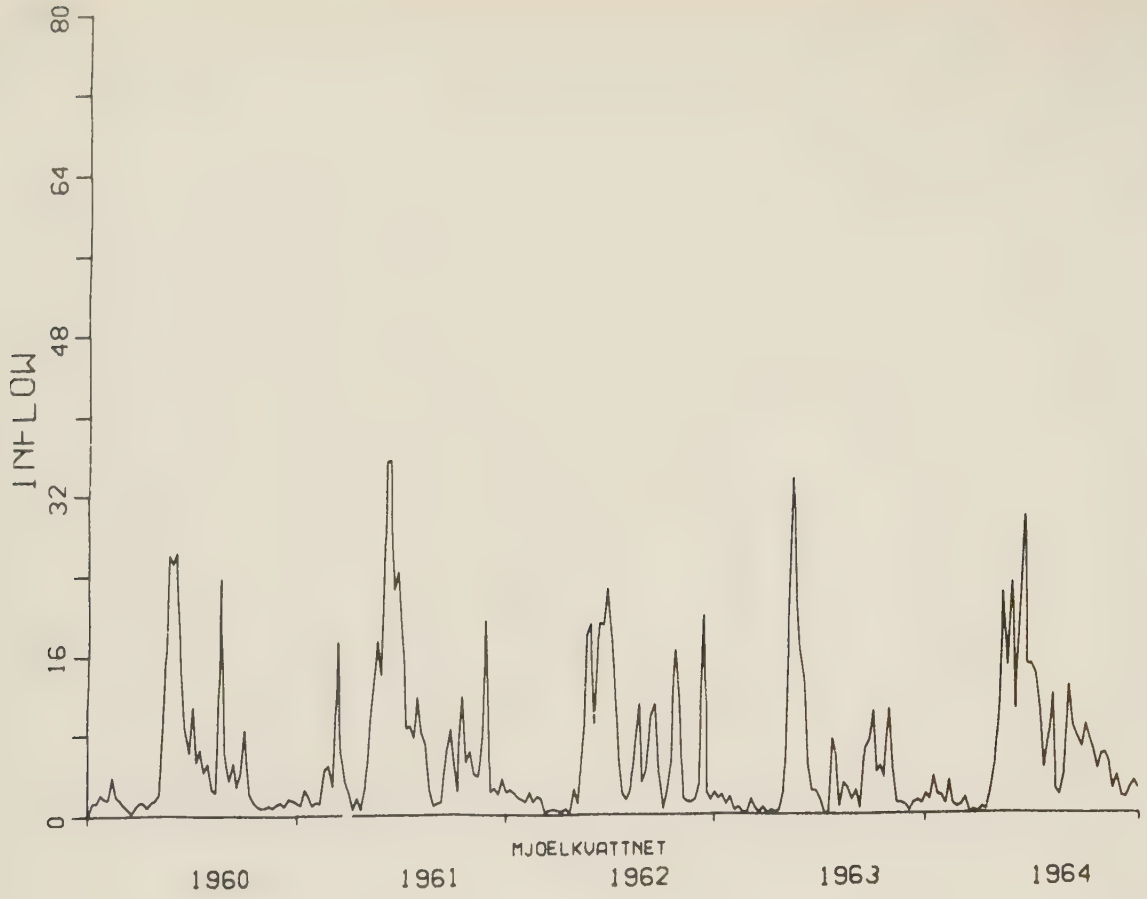

INFLOW
0
8
16
24
32
40
KORSVATTNET
1955
1956
1957
1958
1959

Fig 15 cont.


INFLOW
0
16
32
48
64
80
MJOELKVATTNET
1960
1961
1962
1963
1964

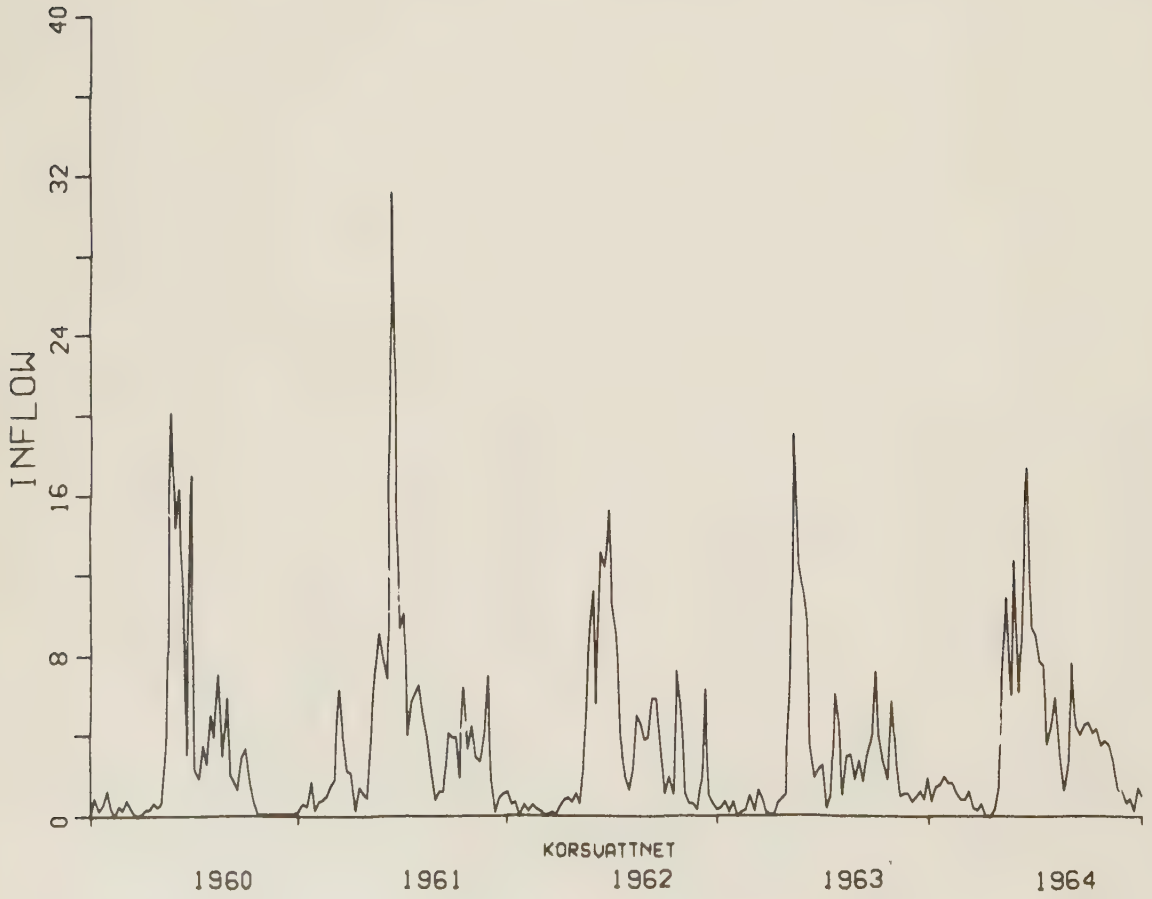
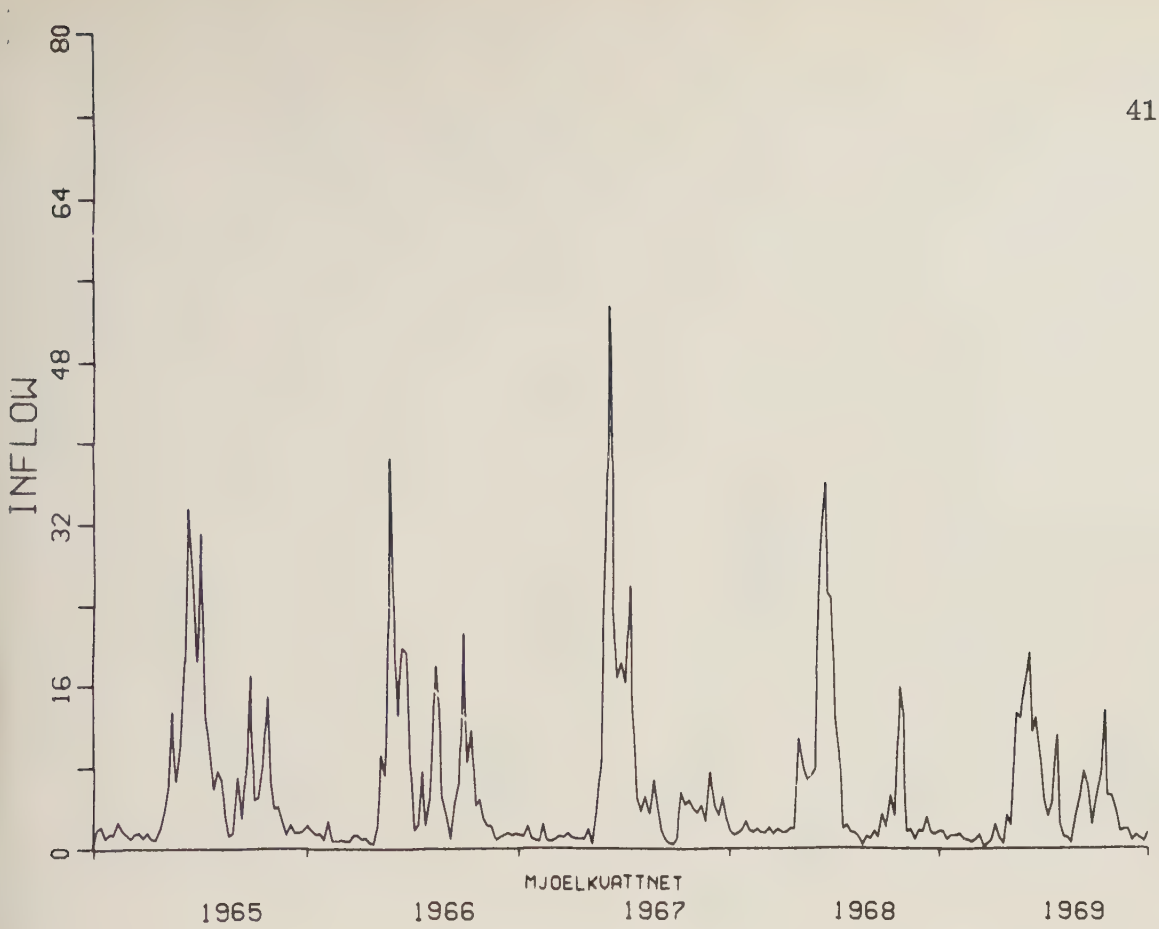

INFLOW
0
8
16
24
32
40
KORSVATTNET
1960
1961
1962
1963
1964

Fig 15 cont.


INFLOW
0
16
32
48
64
80
MJOELKVATTNET
1965
1966
1967
1968
1969

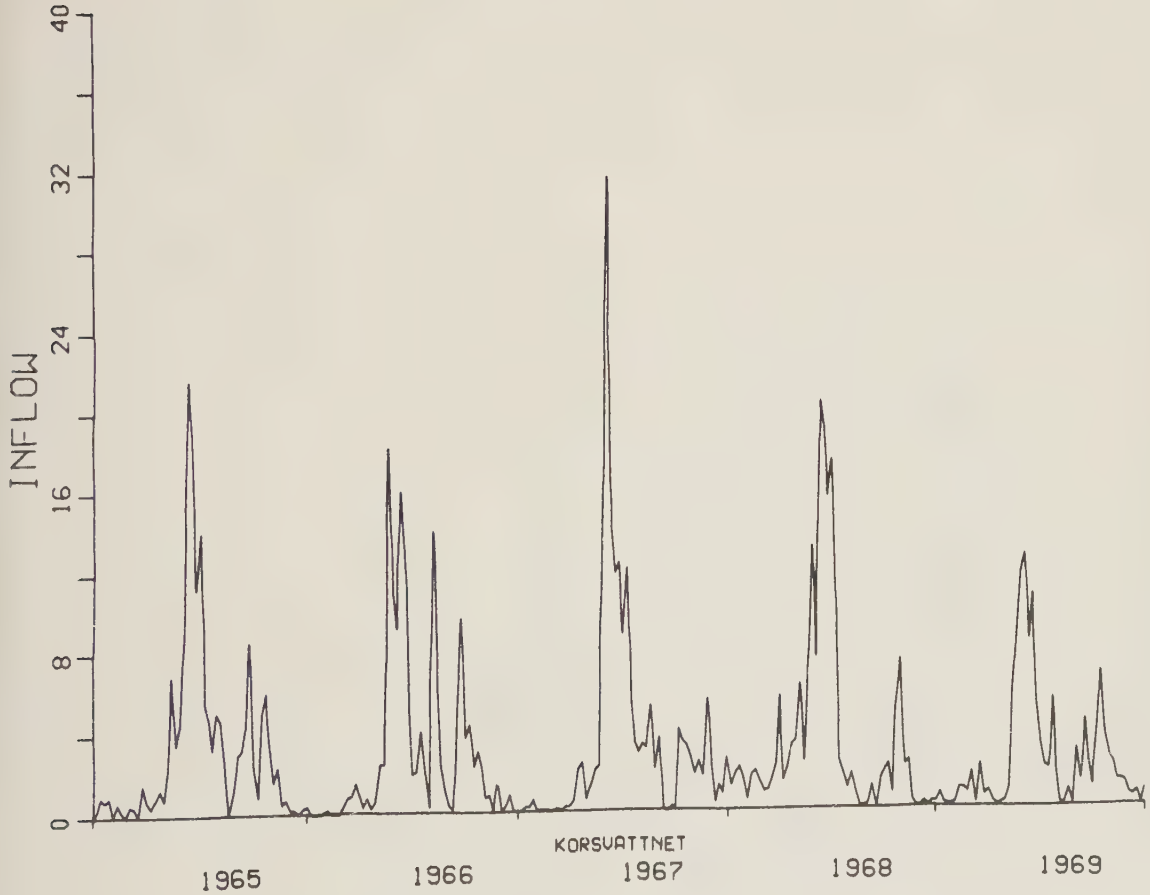
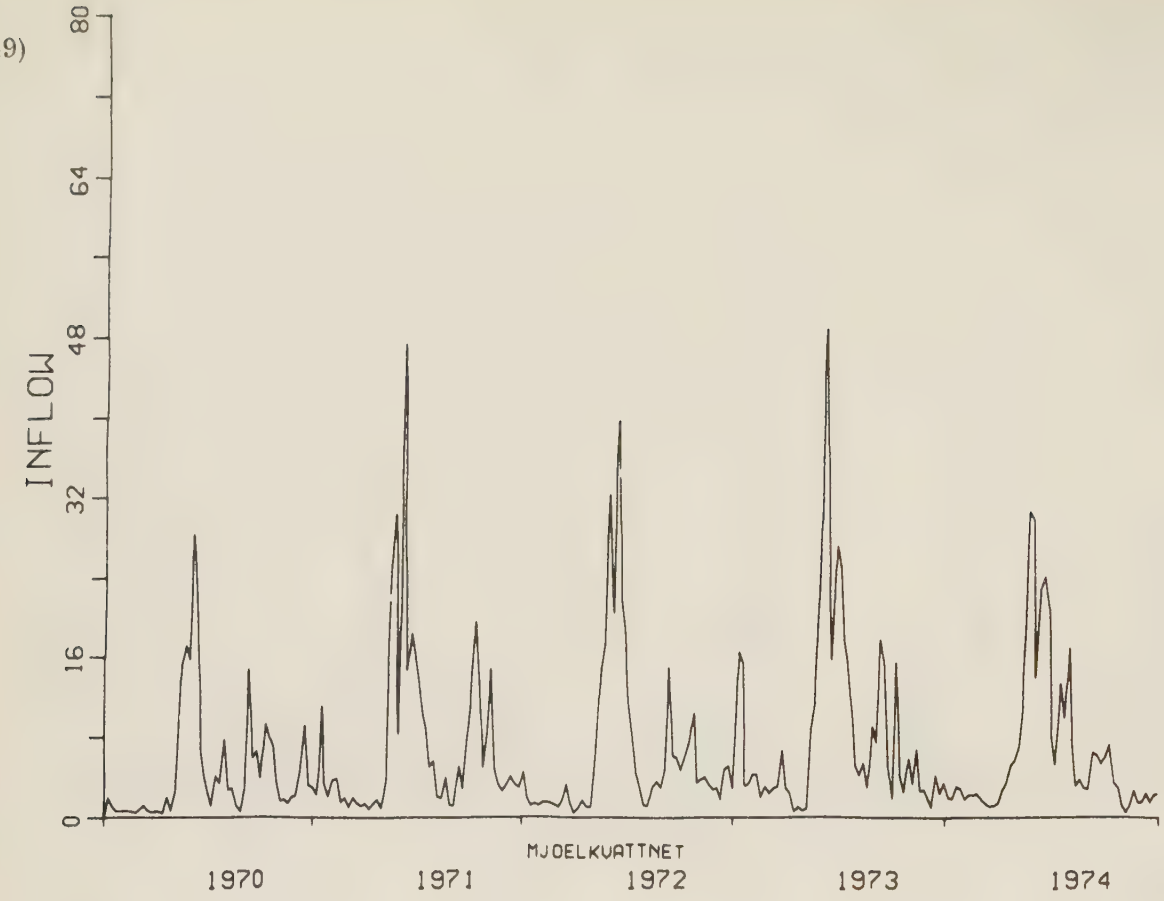

INFLOW
0
8
16
24
32
40
KORSVATTNET
1965
1966
1967
1968
1969

Fig 15 cont.


INFLOW
0
16
32
48
64
80
MJOELKVATTNET
1970
1971
1972
1973
1974

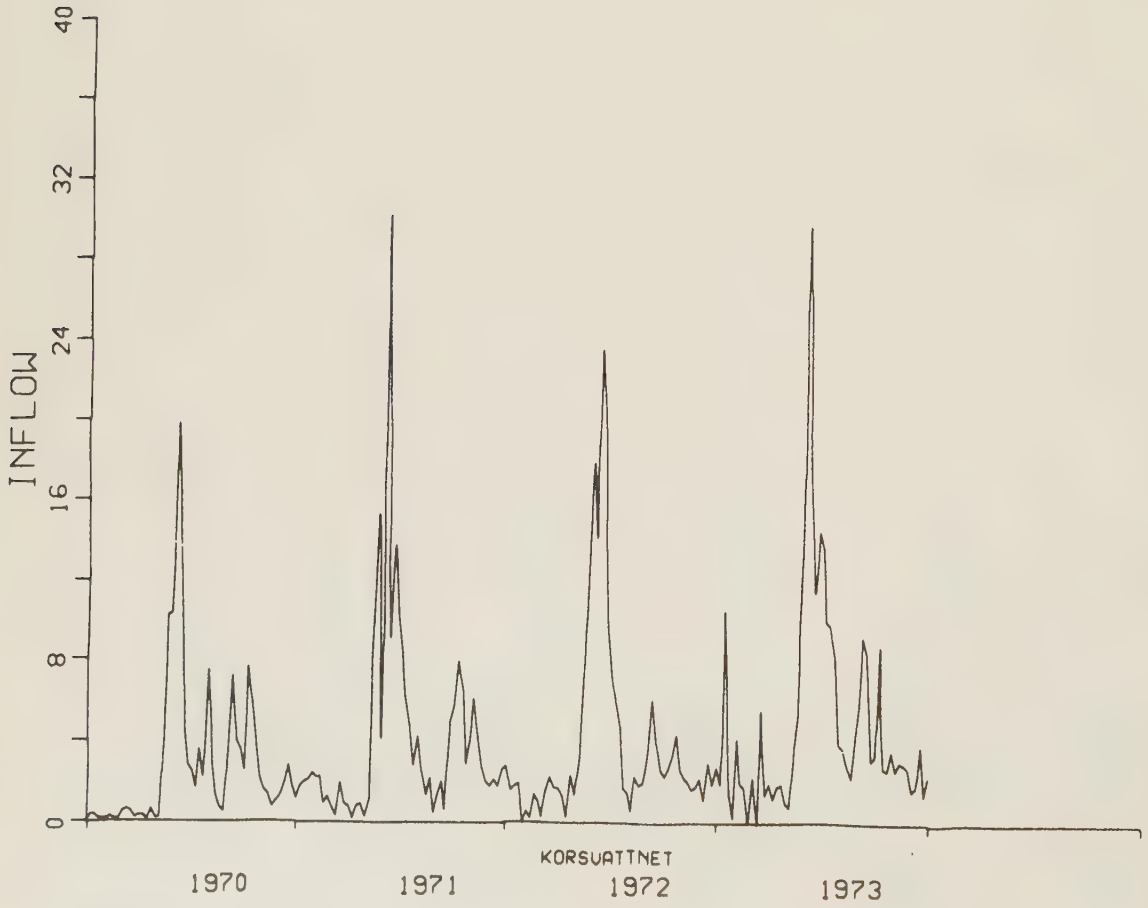

INFLOW
0
8
16
24
32
40
KORSVATTNET
1970
1971
1972
1973

Fig 15 cont.

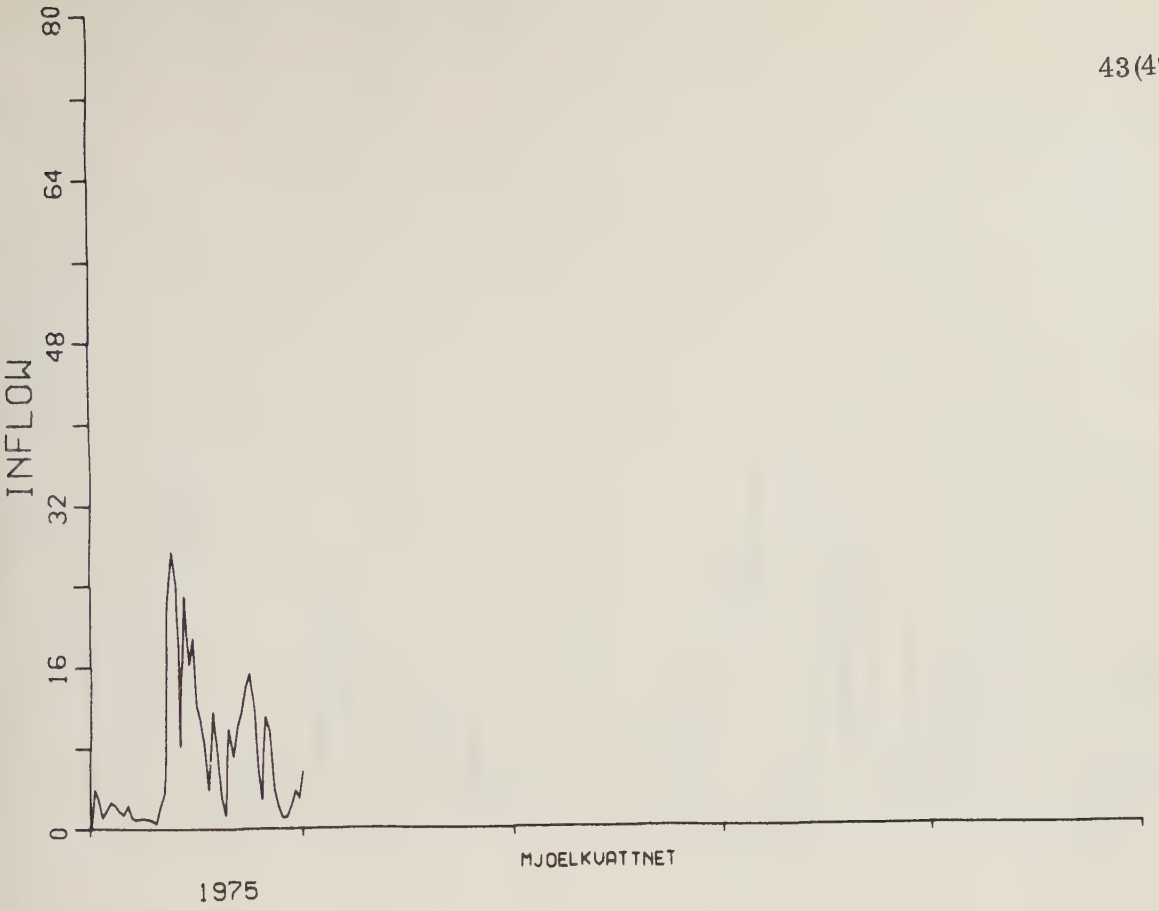


Fig 15 cont.

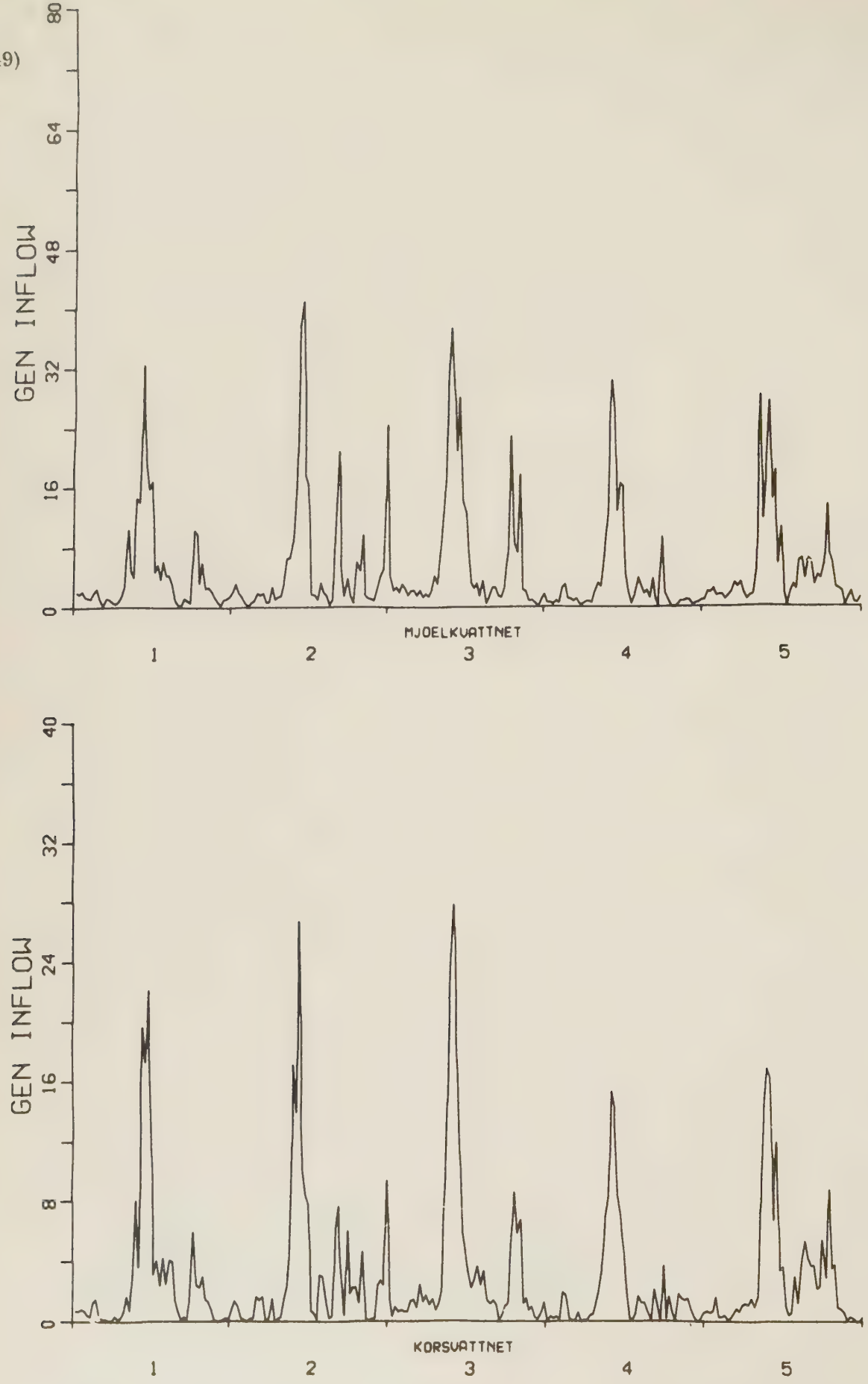
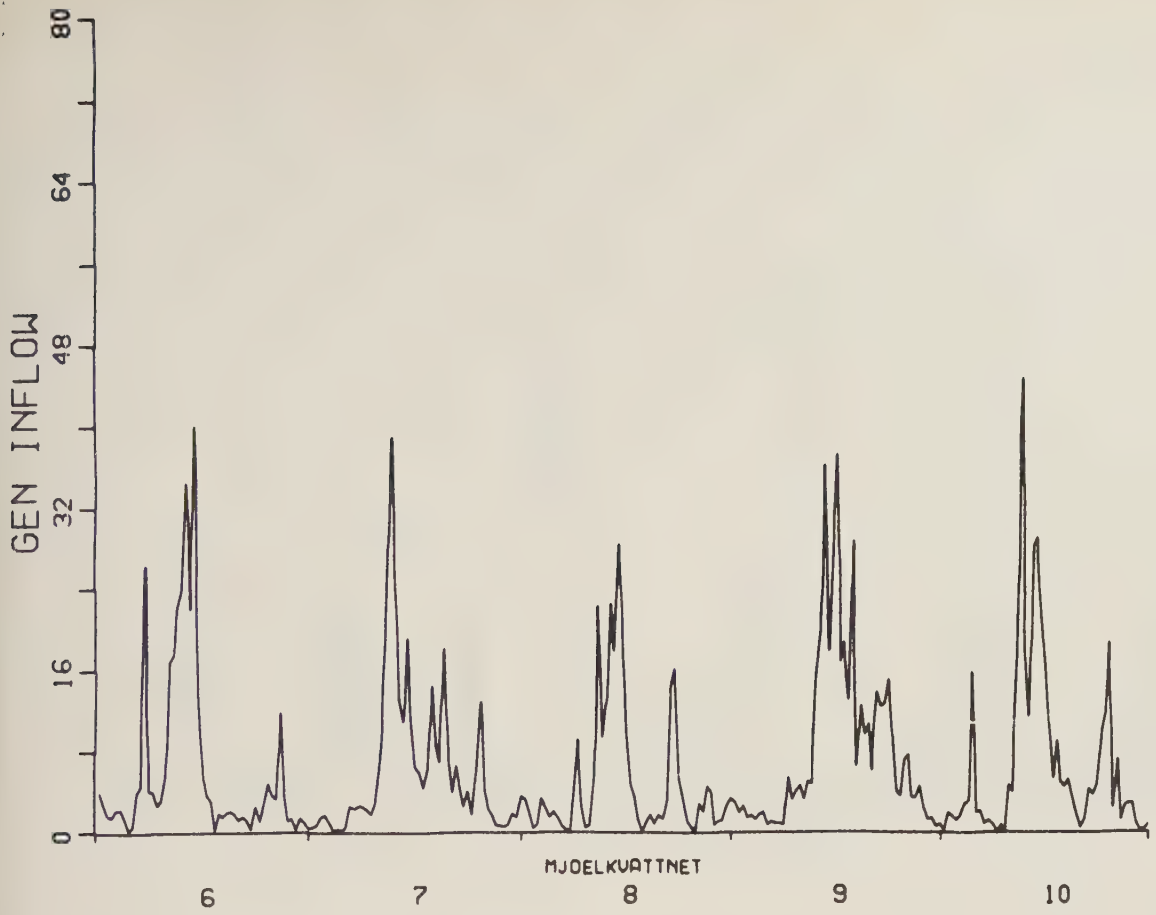


Fig 16 Generated bivariate series nr. 2 of inflows to Stora Mjölkvattnet.
Unit: VE


GEN INFLOW
0
16
32
48
64
80
MJOELKVATTNET
6
7
8
9
10

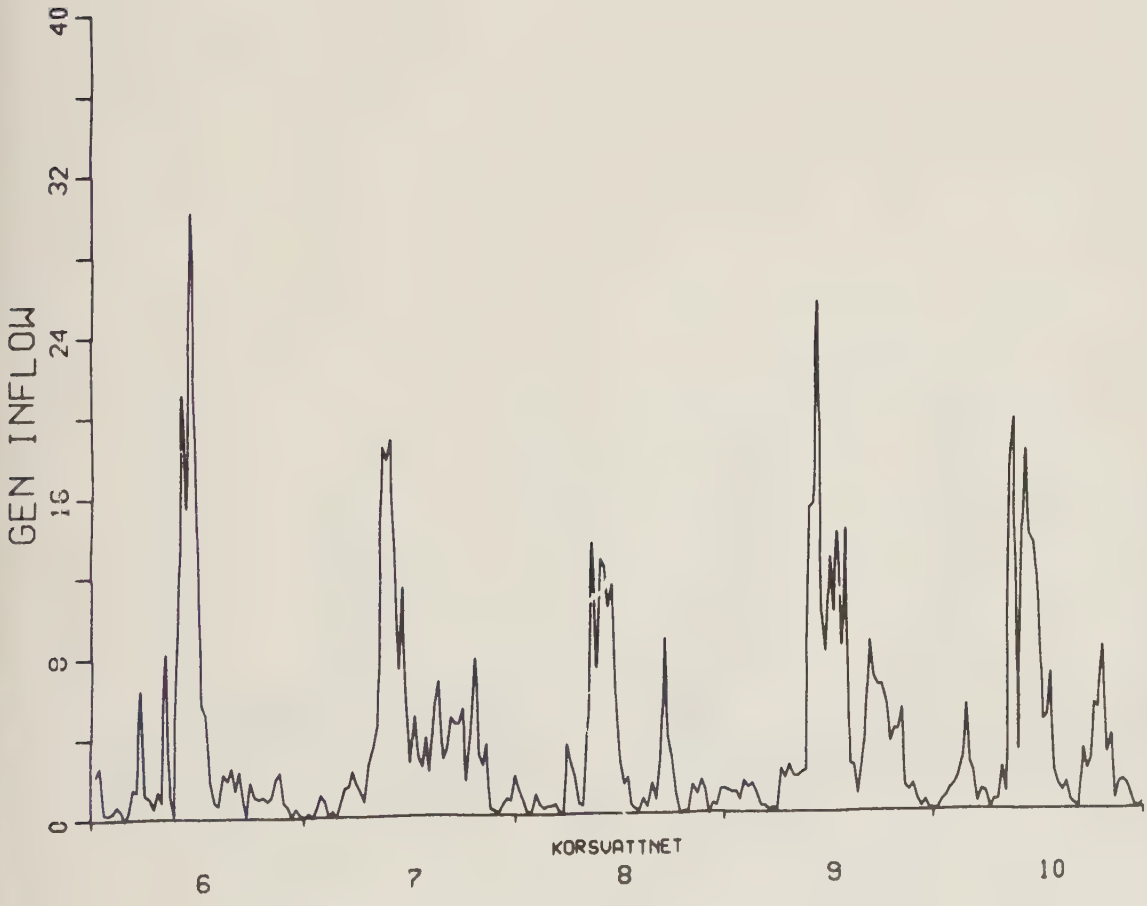
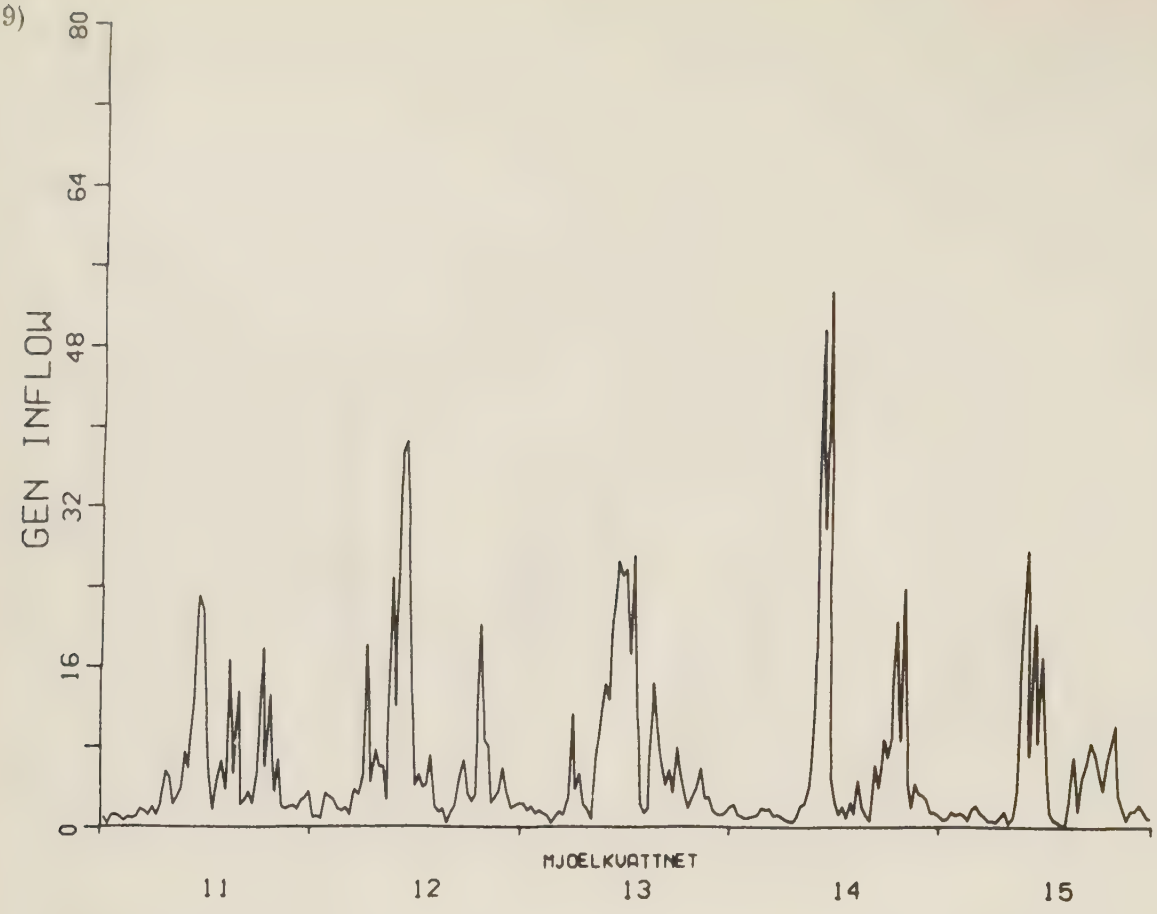

GEN INFLOW
0
8
16
24
32
40
KORSVATTNET
6
7
8
9
10

Fig 16 cont.


GEN INFLOW
0
16
32
48
64
80
MJOELKVATTNET
11
12
13
14
15

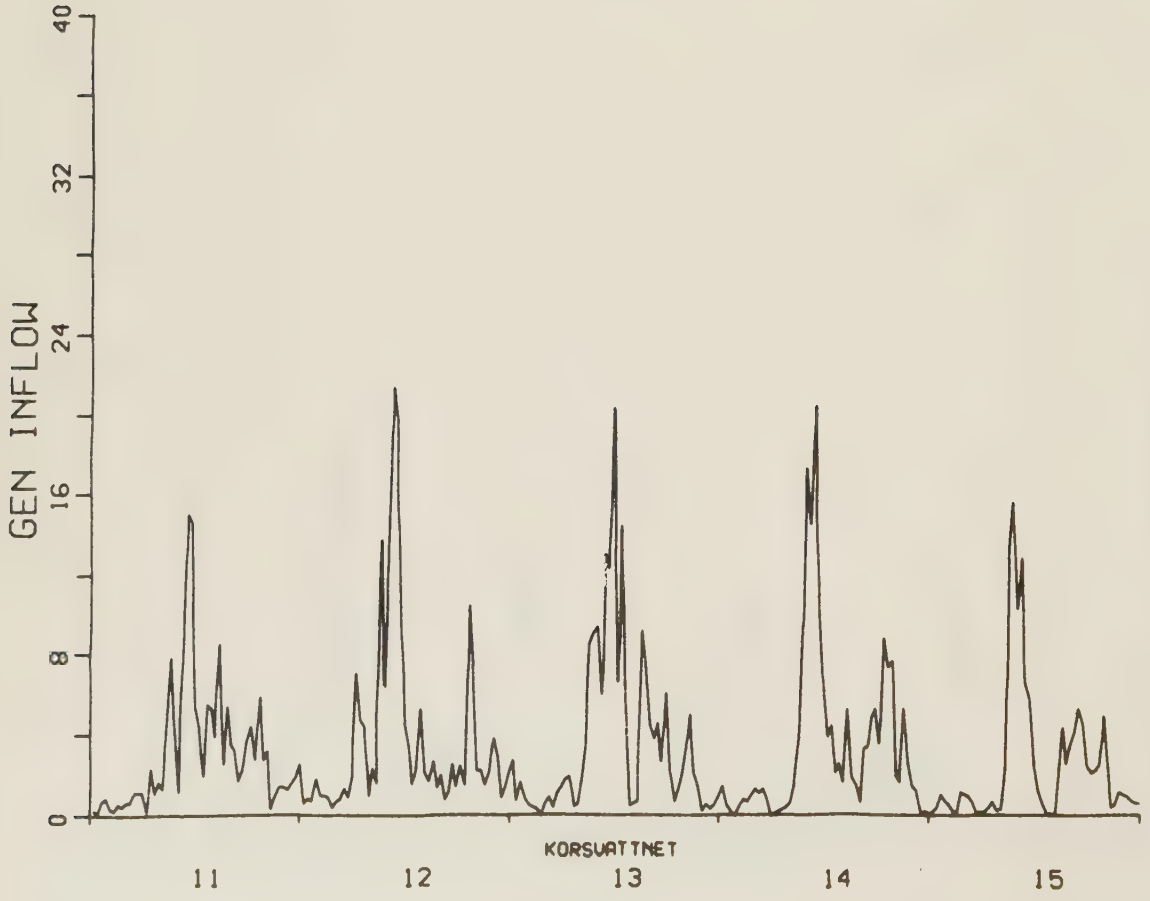
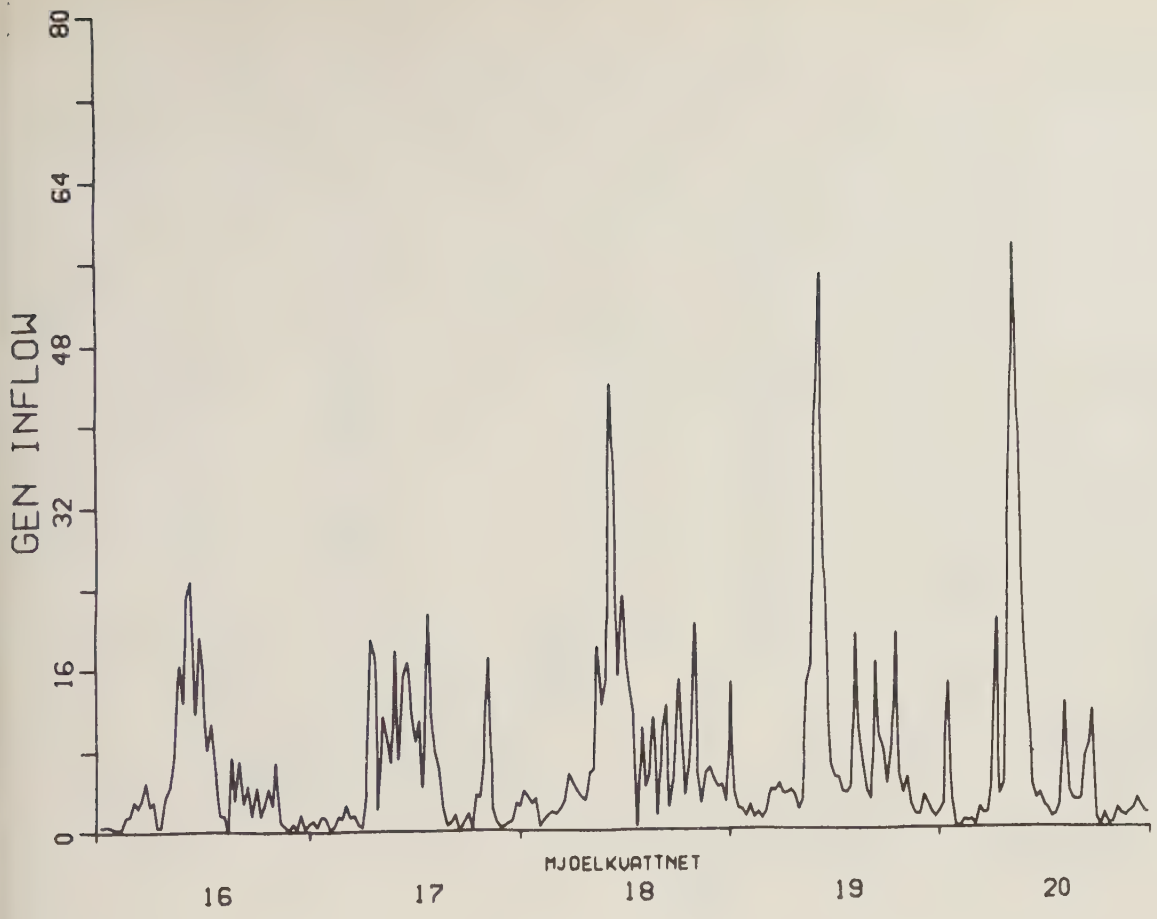

GEN INFLOW
0
8
16
24
32
40
KORSVATTNET
11
12
13
14
15

Fig 16 cont.


GEN INFLOW
0
16
32
48
64
80
MJOELKVATTNET
16
17
18
19
20

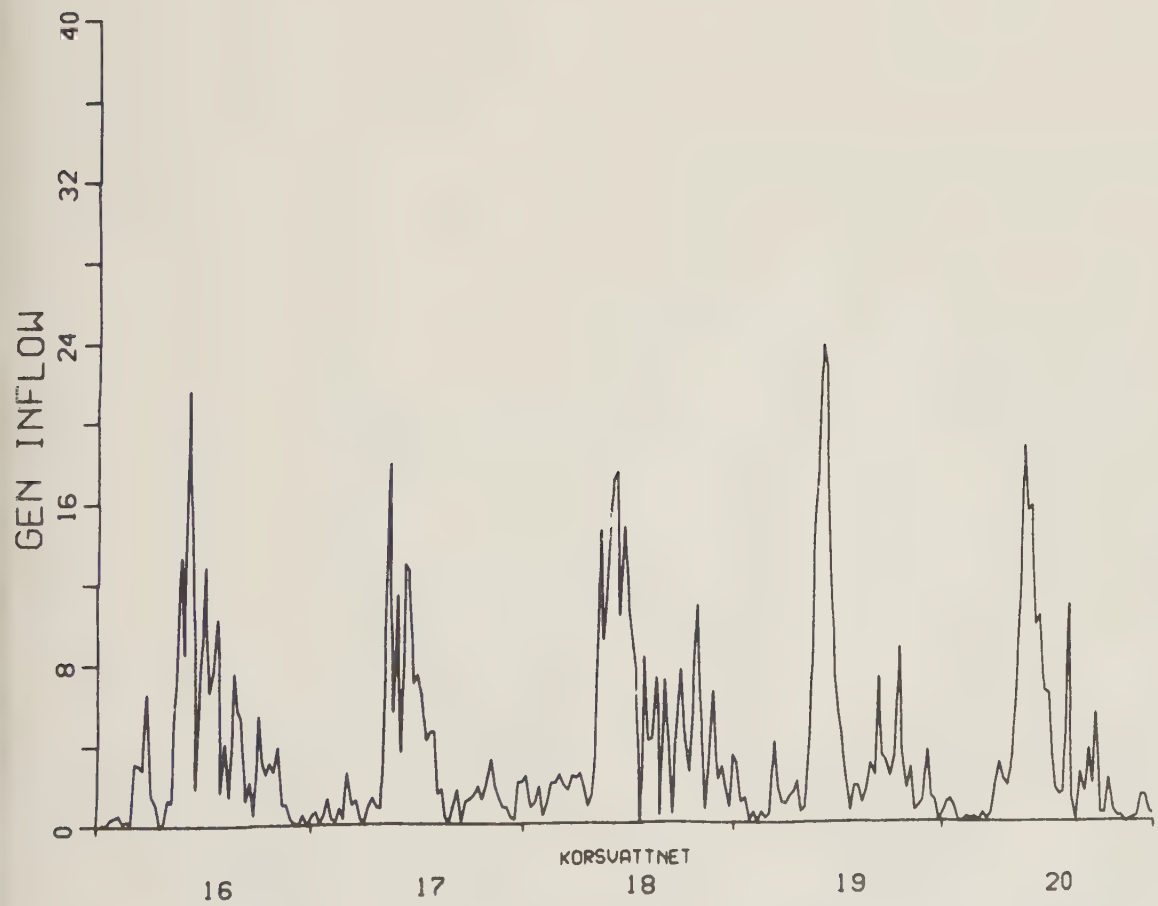
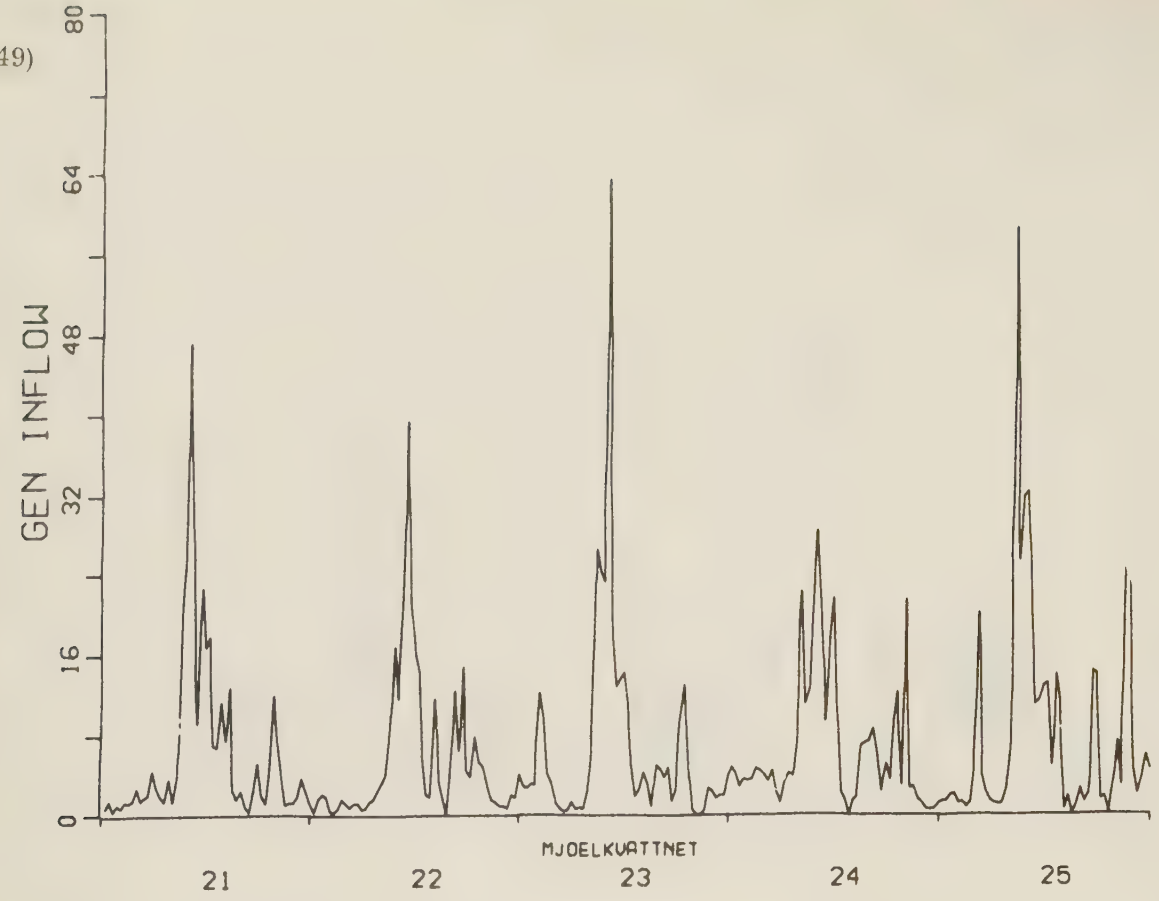

GEN INFLOW
0
8
16
24
32
40
KORSVATTNET
16
17
18
19
20

Fig 16 cont.


GEN INFLOW
0
16
32
48
64
80
MJOELKVATTNET
21
22
23
24
25

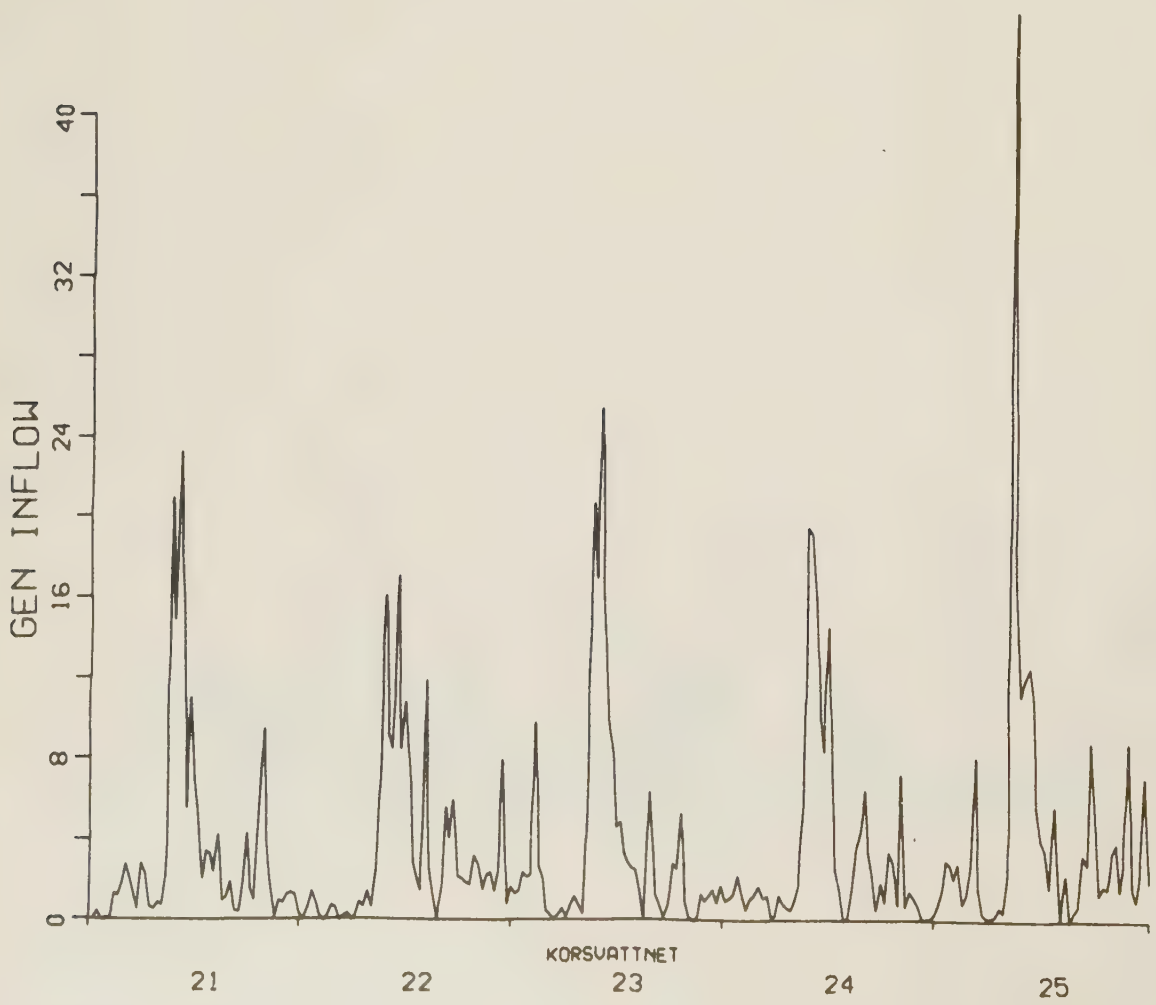
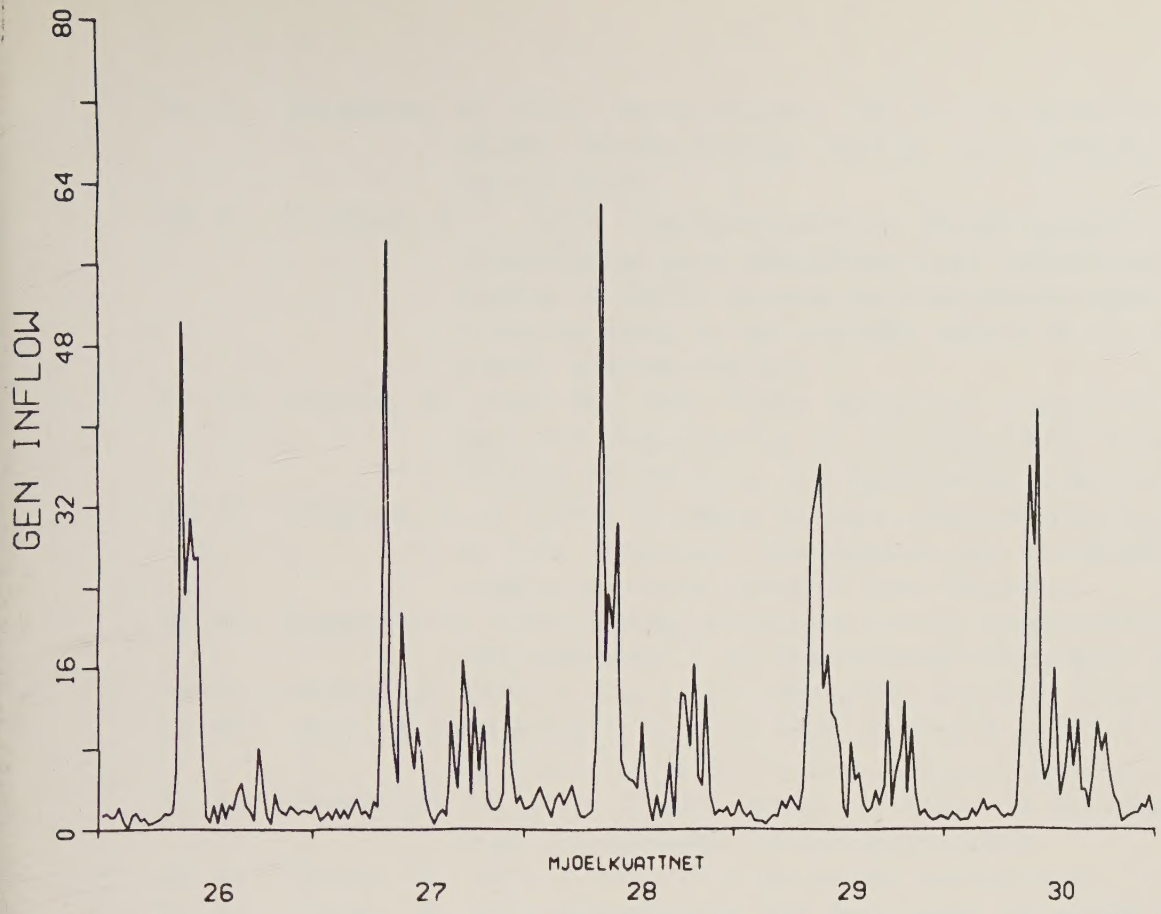

GEN INFLOW
0
8
16
24
32
40
KORSVATTNET
21
22
23
24
25

Fig 16 cont.


GEN INFLOW
0
16
32
48
64
80
MJOELKVATTNET
26
27
28
29
30

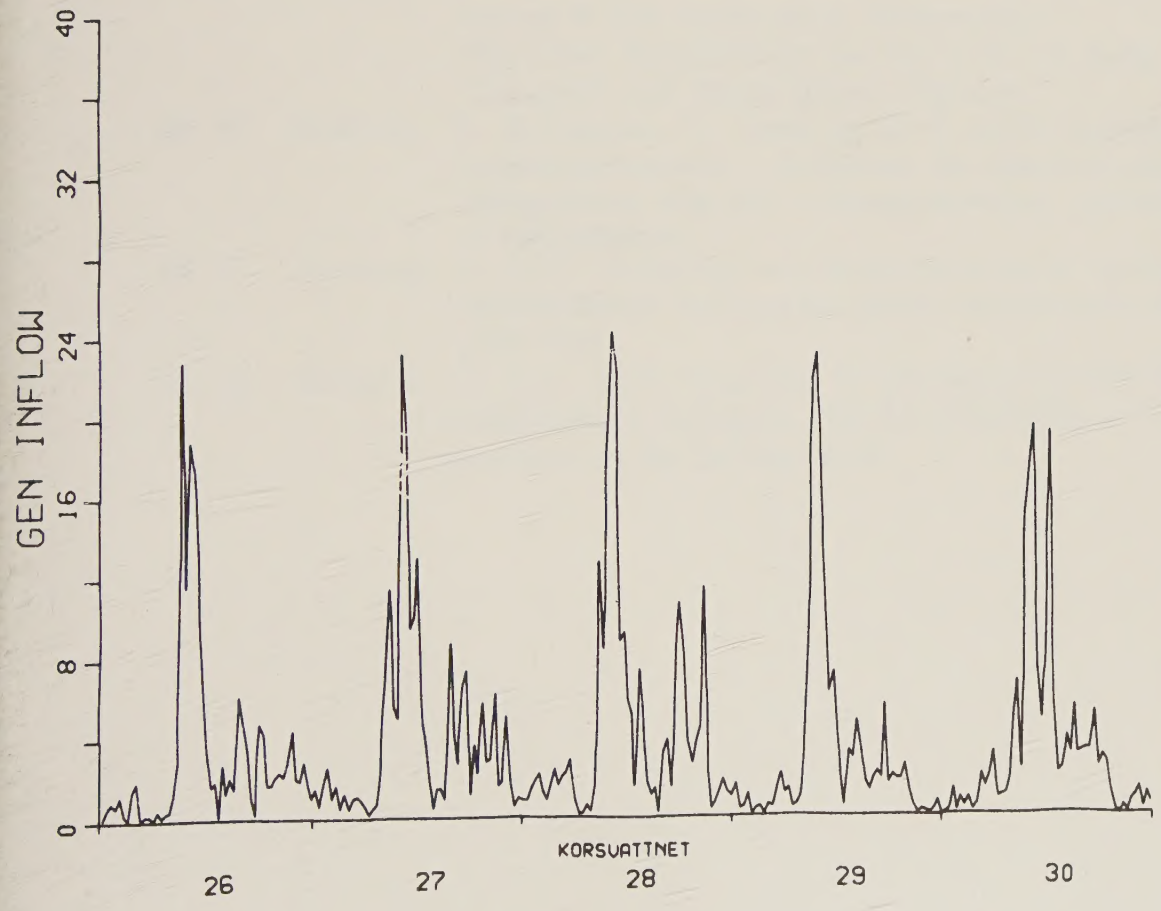

GEN INFLOW
0
8
16
24
32
40
KORSVATTNET
26
27
28
29
30

Fig 16 cont.

Nr 36 Bergqvist, E. 1975: Nåsten-Marsta. Del 5. Lokalklimatet, en metodstudie. Nåsten-Marsta. Part 5. Local climate, a methodological study.

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